STRAIN ENERGY

What is Strain Energy ?

- When a body is subjected to gradual, sudden or impact load, the body deforms and work is done upon it. If the elastic limit is not exceed, this work is stored in the body. This work done or energy stored in the body is called strain energy.

- Energy is stored in the body during deformation process and this energy is called "Strain Energy".

Strain energy = Work done

Resilience :

Total strain energy stored in a body is called resilience.

$$\therefore \mathbf{u} = \frac{\sigma^2}{2E} \times V$$

Where, σ = stress
V = volume of the body

Proof Resilience :

Maximum strain energy which can be stored in a body is called proof resilience.

$$\therefore u_p = \frac{(\sigma_E)^2}{2E} \times V$$

Where, σ_E = stress at elastic limit

Modulus of Resilience :

Maximum strain energy which can be stored in a body per unit volume, at elastic limit is called modulus of resilience.

$$\therefore u_m = \frac{(\sigma_E)^2}{2E}$$

Strain Energy due to Gradual Loading :

- Consider a bar of length L placed vertically and one end of it is attached at the ceiling.

Let P =Gradually applied load

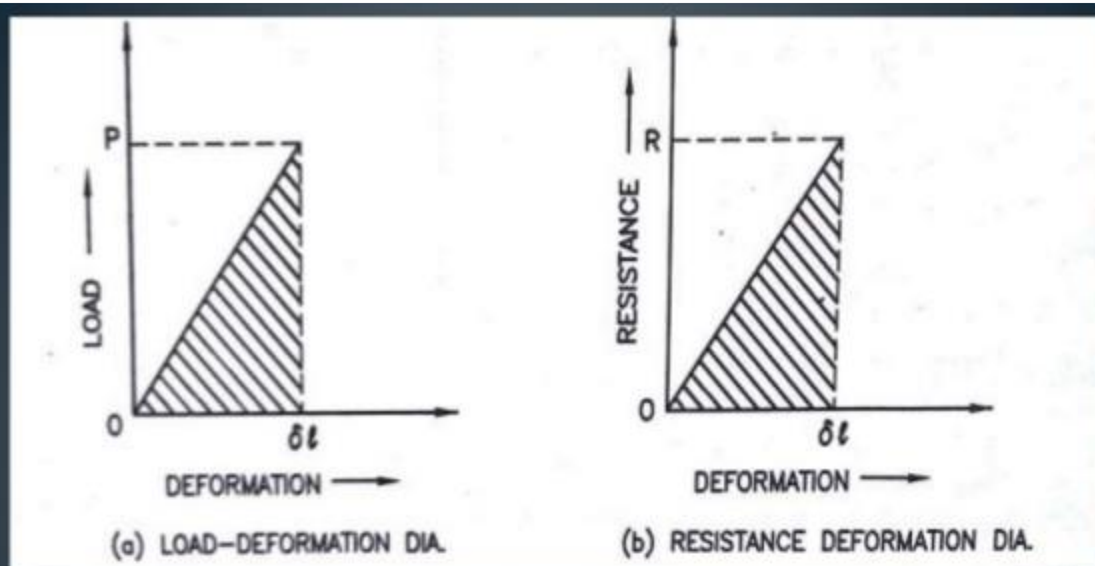
L =length of bar

A =Cross-sectional area of the bar

δl =Deflection produced in the bar

σ =Axial stress induced in the bar. It may be tensile or compressive, depending upon if the bar under consideration is under tensile or compressive load

E =Modulus of elasticity of bar material



Work done on the bar = Area of the load – deformation diagram

$$= \frac{1}{2} \times P \times \delta l \qquad \dots (1)$$

Work Stored in the bar

= Area of the resistance – Deformation diagram

$$= \frac{1}{2} \times R \times \delta l$$

$$= \frac{1}{2} \times (\sigma \times A) \times \delta l \qquad \ldots (2)$$

Now,

Work done = Work stored

$$\therefore \frac{1}{2} P \times \delta l = \frac{1}{2} \sigma \times A \times \delta l$$

$$\therefore P = \sigma \times A$$

$$\therefore \sigma = \frac{P}{A}$$

….. stress due to gradual load.

$$\text{Strain Energy} = \frac{1}{2} \times R \times \delta l$$

$$= \frac{1}{2} \sigma \times A \times \delta l$$

$$= \frac{1}{2} \sigma \times A \times \varepsilon \times l$$

$$= \frac{1}{2} \sigma \times A \times \frac{\sigma}{E} \times l$$

$$= \frac{\sigma^2}{2E} \times A \times l$$

$$R = \sigma \times A$$

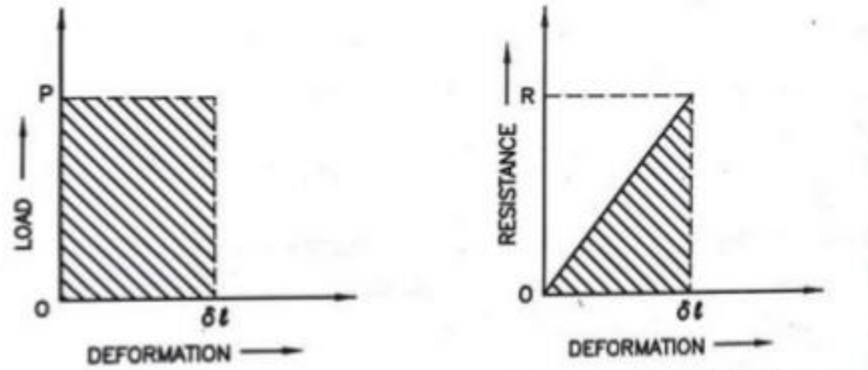
$$\varepsilon = \frac{\delta l}{l}$$

$$E = \frac{\sigma}{\varepsilon}$$

$$u = \frac{\sigma^2}{2E} \times V$$

… strain energy due to gradual loa

Strain Energy due to sudden loading :



(a) LOAD–DEFORMATION DIA.   (b) RESISTANCE DEFORMATION DIA.

- When the load is applied suddenly the value of the load is P throughout the deformation.
- But, Resistance R increase from O to R

Work done on the bar $= P \times \delta l$ ... (1)

$$\text{Work stored in the bar} = \frac{1}{2} \times R \times \delta l$$

$$= \frac{1}{2} \times \sigma \times A \times \delta l \qquad ...(2)$$

Now,

Work done = Work stored

$$\therefore\ P \times \delta l = \frac{1}{2} \times \sigma \times A \times \delta l$$

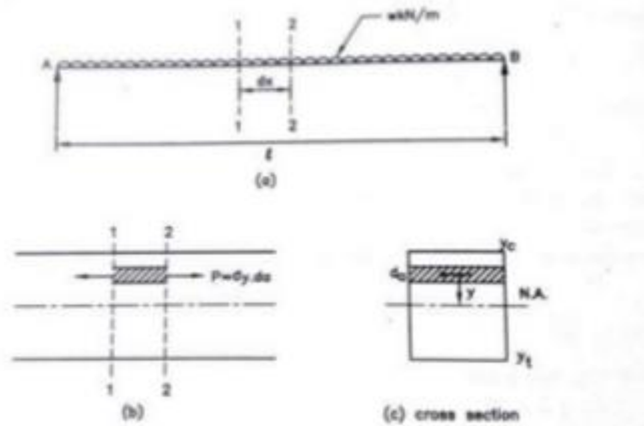
$$\therefore\ P = \frac{1}{2} \times \sigma \times A$$

$$\therefore\ \boxed{\sigma = \frac{2P}{A}}$$

- Hence , the Maximum Stress intensity due to a suddenly applied load is **Twice** the stress intensity produced by the load of the same magnitude applied gradually.

Strain Energy due to bending (flexure) :

- Consider two transverse section 1-1 and 2-2 of a beam distant dx apart as shown in fig.



- Consider a small strip of area **da** at distant **y** from the neutral axis.
- B.M. in small portion dx will be constant.

$$\frac{M}{I} = \frac{\sigma}{y}$$

$$\therefore \ \sigma = \frac{M}{I} \times y \qquad \dots (1)$$

$\therefore$ Strain energy stored in small strip of area **da.**

$$u = \frac{\sigma^2}{2E} \times v$$

$$= \frac{\sigma^2}{2E} \times (da \cdot dx)$$

$$= \frac{1}{2E} \times (\frac{M}{I} \cdot y)^2 \times (da \cdot dx)$$

$$= \frac{1}{2E} \cdot \frac{M^2 \cdot y^2}{I^2} \times da \cdot dx \qquad \ldots(2)$$

$\therefore$ Starin energy stored in entire section of a beam.

$$u_{total} = \sum_{y=y_t}^{y=y_c} \frac{1}{2E} \cdot \frac{M^2 \cdot y^2}{I^2} \cdot da \cdot dx$$

$$= \frac{1}{2E} \cdot \frac{M^2 \cdot dx}{I^2} \sum_{y_t}^{y_c} y^2 \cdot da$$

$$= \frac{1}{2E} \cdot \frac{M^2 \cdot dx}{I^2} I$$

$\sum y^2 \cdot da = I$

= second moment of area.

$$u_{total} = \frac{M^2}{2EI} dx \qquad \ldots(3)$$

- Now, for strain energy in entire beam, integrate between limits 0 to l.

$$\therefore\ u = \int_0^l \frac{M^2}{2EI} \cdot dx$$

... Strain energy due to bending.

examples

- Ex-1 :

An axial pull of 50 kN is suddenly applied to a steel bar 2m long and 1000 mm^2 in cross section. If modulus of elasticity of steel is 200 kN/ mm^2.

Find, (i) maximum instantaneous stress

(ii) maximum instantaneous extension

(iii) Strain energy

(iv) modulus of resilience.

Solution :

here, P = 50 kN (Sudden load)

$A = 1000 \text{ mm}^2$

$l = 2m = 2000 \text{ mm}$

$E = 200 \text{ kN/mm}^2 = 200 \times 10^3 \text{ N/mm}^2$

(i) Maximum instantaneous stress :

$$\sigma = \frac{2P}{A} = \frac{2\times50\times10^3}{1000} \boxed{= 100\ \text{N/mm}^2}$$

(ii) Maximum instantaneous extension :

$$E = \frac{\sigma}{\varepsilon}$$

$$\varepsilon = \frac{\sigma}{E} = \frac{100}{200\times10^3} = 5 \times 10^{-4}$$

$$\varepsilon = \frac{\delta l}{l}$$

$$\delta l = \varepsilon \cdot l$$

$$= 5 \times 10^{-4} \times 2000$$

$$\boxed{= 1\ \text{mm}}$$

(iii) Strain energy (u) :

$$u = \frac{\sigma^2}{2E} \times V$$

$$= \frac{(100)^2}{2 \times 200 \times 10^3} \times (1000 \times 2000)$$

$$\boxed{= 50{,}000 \ N \cdot mm}$$

(iv) Modulus of resilience (u_m) :

$$u_m = \frac{\sigma^2}{2E}$$

$$= \frac{(100)^2}{2 \times 200 \times 10^3}$$

$$\boxed{= 0.025 \ N \cdot mm/mm^3}$$