VIRTUAL WORK

OBJECTIVES:

- Principle of virtual work and applies to determining the equilibrium configuration of a series of pin-connected members
- Establish the potential energy function and use the potential energy method

Chapter Outline

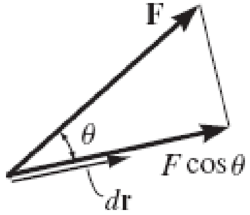
1. Definition of Work
2. Principle of Virtual Work
3. Principle of Virtual Work for a System of Connected Rigid Bodies
4. Conservative Forces
5. Potential Energy
6. Potential-Energy Criterion for Equilibrium
7. Stability of Equilibrium Configuration

11.1 Definition of Work

Work of a Force

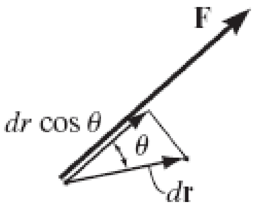
- In mechanics, a force **F** does work only when it undergoes a displacement in the direction of the force
- Consider the force **F** located in the path s specified by the position vector **r**
- Work dU is a scalar quantity defined by the dot product

$$dU = \mathbf{F} \cdot d\mathbf{r}$$



- If the angle between the tails of d**r** and **F** is θ,

$$dU = F\, ds \cos \theta$$



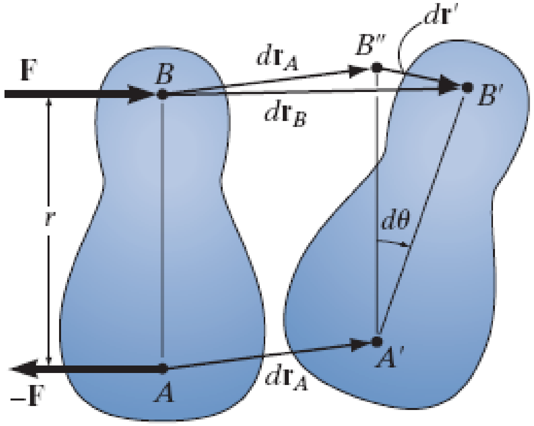
11.1 Definition of Work

Work of a Couple Moment

- When the body translates such that the component of displacement of the body along the line of action of each force is ds_t
- Positive work ($F\ ds_t$) cancels negative work of the other ($-F\ ds_t$)
- For work of both forces,

$$dU = F(r/2)\ d\theta + F(r/2)\ d\theta = (Fr)\ d\theta$$

$$dU = M\ d\theta$$



11.1 Definition of Work

Virtual Work

- For virtual work done by a force undergoing virtual displacement,

$$\delta U = F \cos\theta \, \delta s$$

- When a couple undergoes a virtual rotation in the plane of the couple forces, for virtual work,

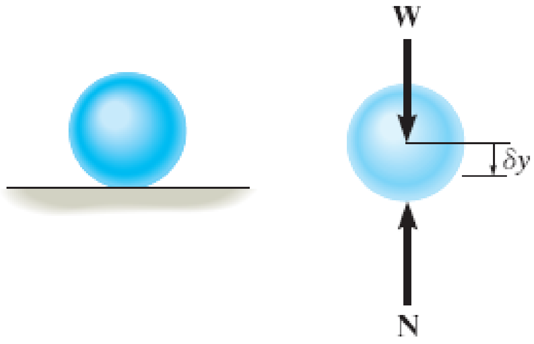
$$\delta U = M \, \delta\theta$$

11.2 Principle of Virtual Work

- Consider the FBD of the ball which rests on the floor
- Imagine the ball to be displacement downwards a virtual amount δy and weight does positive virtual work W δy and normal force does negative virtual work -N δy
- For equilibrium,

$$\delta U = W\delta y - N\delta y = (W-N)\delta y = 0$$

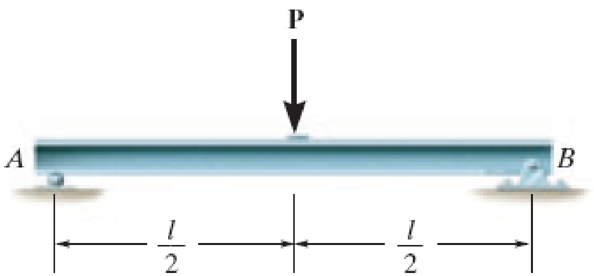
- Since δy ≠ 0, then N = W

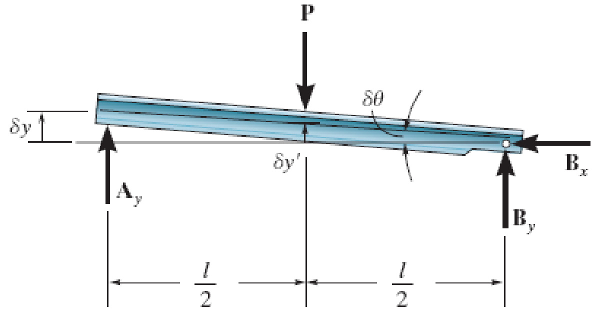


11.2 Principle of Virtual Work

- Consider simply supported beam, with a given rotation about point B
- Only forces that do work are **P** and $\mathbf{A}_y$
- Since $\delta y = l\delta\theta$ and $\delta y' = (l/2)\delta\theta$, virtual work

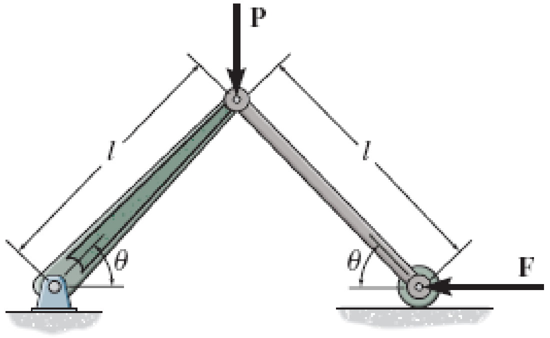
  $\delta U = A_y(l\delta\theta) - P(l/2)\delta\theta = (A_y - P/2)l\ \delta\theta = 0$
- Since $\delta\theta \neq 0$, $A_y = P/2$
- Excluding $\delta\theta$, terms in parentheses represent moment equilibrium about B

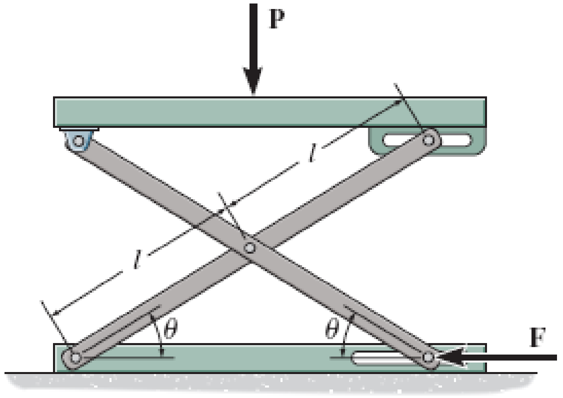




11.3 Principle of Virtual Work for a System of Connected Rigid Bodies

- Method of virtual work used for solving equilibrium problems involving a system of several connected rigid bodies
- Before applying, specify the number of degrees of freedom for the system and establish the coordinates that define the position of the system





11.3 Principle of Virtual Work for a System of Connected Rigid Bodies

Procedure for Analysis

Free Body Diagram

- Draw FBD of the entire system of connected bodies and sketch the independent coordinate q
- Sketch the deflected position of the system on the FBD when the system undergoes a positive virtual displacement δq

11.3 Principle of Virtual Work for a System of Connected Rigid Bodies

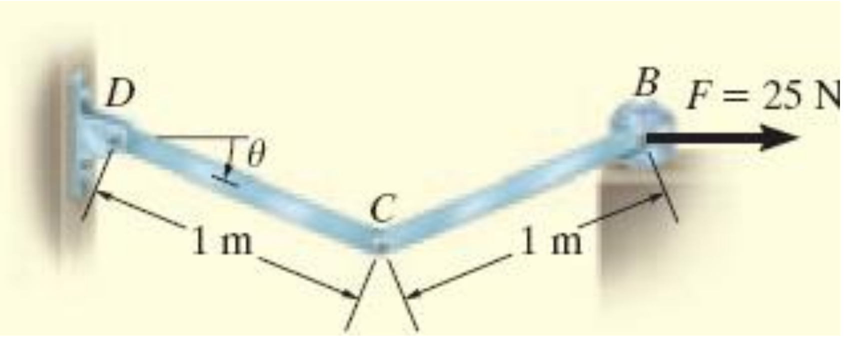
Procedure for Analysis

Virtual Displacements

- Indicate position coordinates s_i,
- Each coordinate system should be parallel to line of action of the active force
- Relate each of the position coordinates s_i to the independent coordinate q, then differentiate for virtual displacements δs_i in terms of δq
- n virtual work equations can be written, one for each independent coordinate

Example 11.1

Determine the angle θ for equilibrium of the two-member linkage. Each member has a mass of 10 kg.



Solution

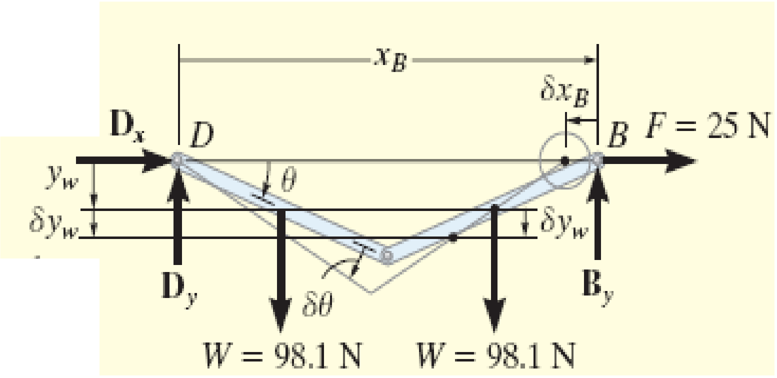
FBD

One degree of freedom since location of both links may be specified by a single independent coordinate.

θ undergoes a positive (CW) virtual rotation δθ, only the active forces, **F** and the 2 9.81N weights do work.

$$x_B = 2(1 \cos \theta) m \qquad \delta x_B = -2 \sin \theta \delta\theta \ m$$

$$y_w = \frac{1}{2}(1 \sin \theta) m \qquad \delta y_w = 0.5 \cos \theta \delta\theta \ m$$



Solution

Virtual Work Equation

If δx_B and δy_w were both positive, forces **W** and **F** would do positive work.

For virtual work equation for displacement $\delta\theta$,

$$\delta U = 0; \qquad W\delta y_w + W\delta y_w + F\delta x_B = 0$$

Relating virtual displacements to common $\delta\theta$,

$$98.1(0.5\cos\theta\ \delta\theta) + 9.81(0.5\cos\theta\ \delta\theta) + 25(-2\sin\theta\ \delta\theta) = 0$$

Since $\delta\theta \neq 0$,

$$(98.1\cos\theta - 50\sin\theta)\ \delta\theta = 0$$

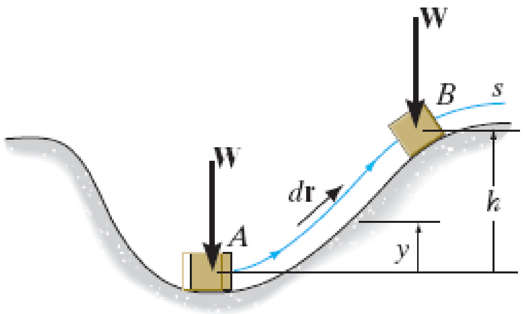
$$\theta = \tan^{-1}(9.81/50) = 63.0^\circ$$

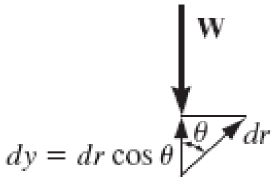
11.4 Conservative Forces

Weight

- Consider a block of weight that travels along the path
- If the block moves from to , through the vertical displacement , the work is

$$U = -\int_0^y W dy = -Wh$$



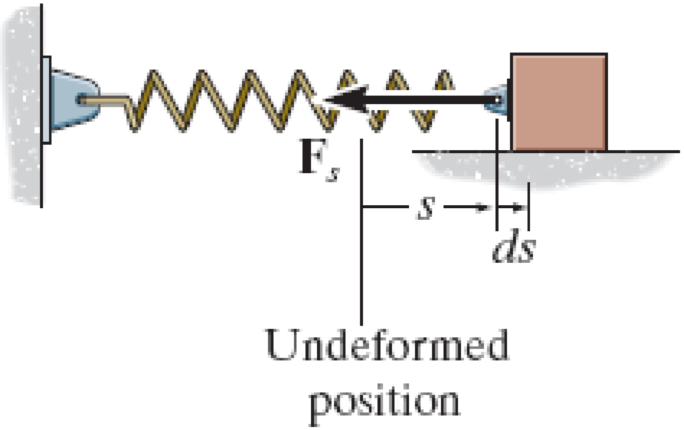


11.4 Conservative Forces

Spring Force

- For either extension or compression, work is independent of the path and is simply

$$U = \int_{s_1}^{s_2} F_s \, ds = \int_{s_1}^{s_2} (-ks)\, ds = -\left(\frac{1}{2} k s_2^2 - \frac{1}{2} k s_1^2 \right)$$

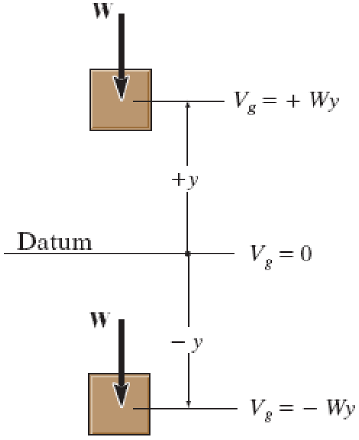


11.5 Potential Energy

Gravitational Potential Energy

- Measuring y as positive upwards, for gravitational potential energy of the body's weight **W,**

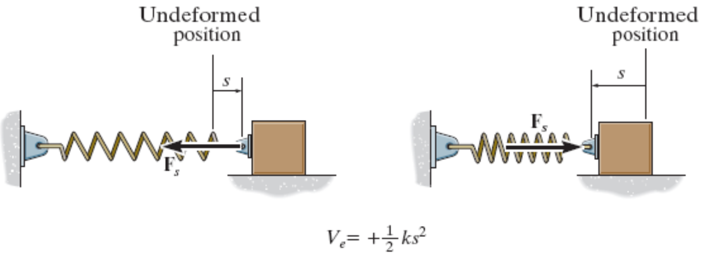
$$V_g = W\ y$$



Elastic Potential Energy

- When spring is elongated or compressed from an undeformed position (s = 0) to a final position s,

$$V_e = ½\ ks2$$



11.5 Potential Energy

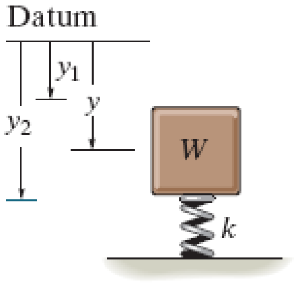
Potential Function

- If a body is subjected to both gravitational and elastic forces, potential energy or potential function V of the body can be expressed as an algebraic sum

$$V = V_g + V_e$$

- Work done by all the conservative forces acting on the system in moving it from q_1 to q_2 is measured by the difference in V

$$U_{1\text{-}2} = V(q_1) - V(q_2)$$



11.6 Potential-Energy Criterion for Equilibrium

System having One Degree of Freedom

$$dV/dq = 0$$

- When a frictionless connected system of rigid bodies is in equilibrium, the first variation or change in V is zero
- Change is determined by taking first derivative of the potential function and setting it to zero

11.7 Stability of Equilibrium Configuration

Stable Equilibrium

- *Stable* system has a tendency to return to its original position

Neutral Equilibrium

- A *neutral equilibrium* system still remains in equilibrium when the system is given a small displacement away from its original position.

Unstable Equilibrium

- An *unstable* system has atendency to be *displaced further away* from its original equilibrium position when it is given a small displacement

11.7 Stability of Equilibrium Configuration