Deflection
Energy Method

Energy Method

External Work

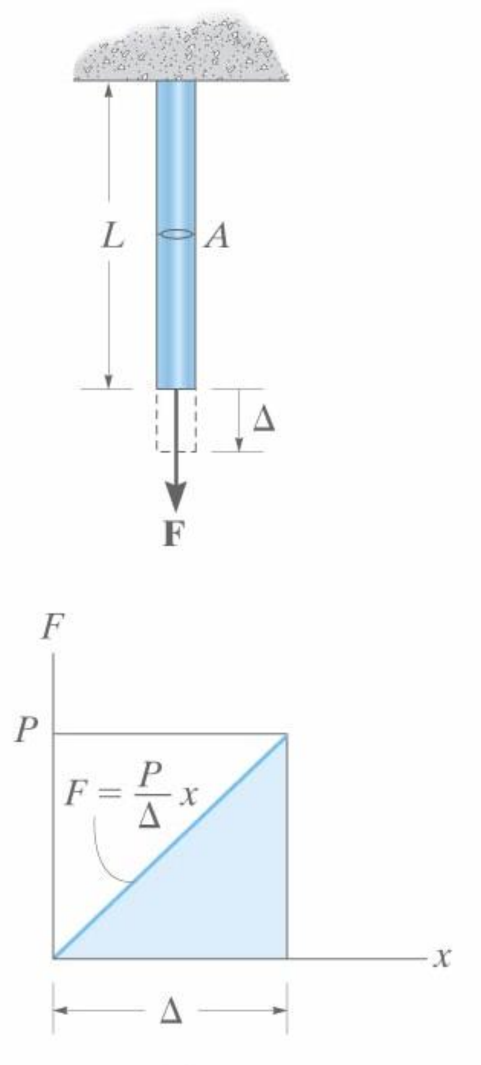
When a force F undergoes a displacement *dx* in the same direction as the force, the work done is $dU_e = F\,dx$

If the total displacement is x the work become

$$U_e = \int_0^x F\,dx$$

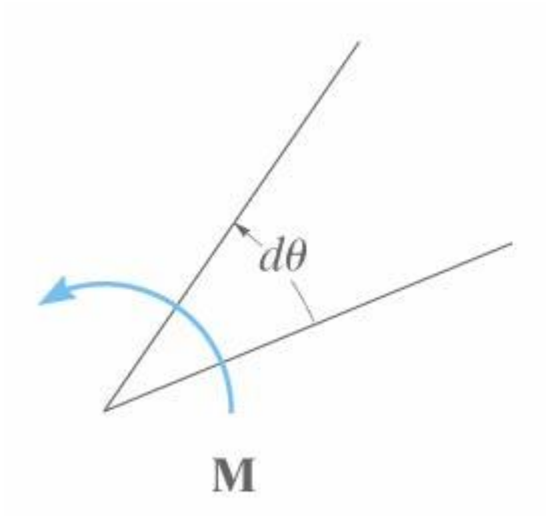
$$U_e = \tfrac{1}{2} P\Delta$$

The force applied gradually



(a)

The work of a moment is defined by the product of the magnitude of the moment **M** and the angle $d\theta$ then



$$dU_e = M \, d\theta$$

If the total angle of rotation is θ the work become:

$$U_e = \int_0^{\theta} M \, d\theta$$

$$U_e = \tfrac{1}{2} M \theta$$

The moment applied gradually

Energy Method

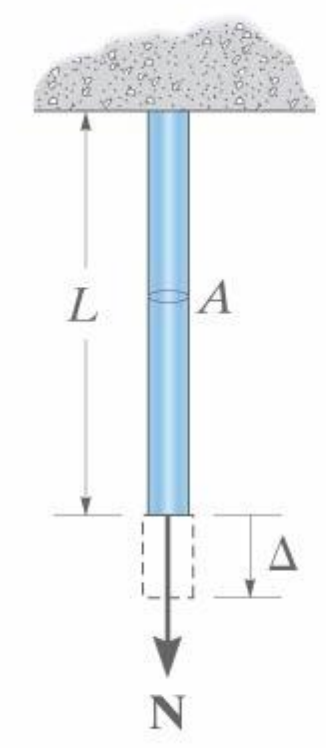
Strain Energy – Axial Force

$$\sigma = E\varepsilon$$

$$\sigma = \frac{N}{A}$$

$$\varepsilon = \frac{\Delta}{L}$$

$$\Rightarrow \quad \Delta = \frac{NL}{AE}$$



$$P = N$$

$$U = \tfrac{1}{2} P \Delta$$

$$U_i = \frac{N^2 L}{2AE}$$

N = internal normal force in a truss member caused by the real load

L = length of member

A = cross-sectional area of a member

E = modulus of elasticity of a member

Energy Method

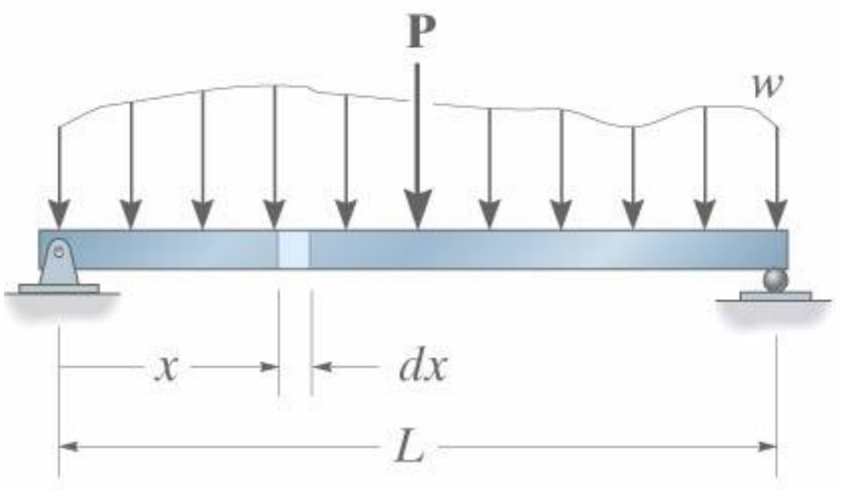
Strain Energy – Bending

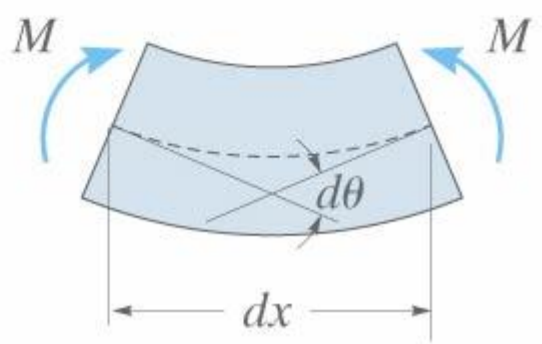
$$d\theta = \frac{M}{EI}dx$$

$$U = \tfrac{1}{2} M\theta$$

$$dU_i = \frac{M^2 dx}{2EI}$$

$$U_i = \int_0^L \frac{M^2 dx}{2EI}$$





Principle of Virtual Work

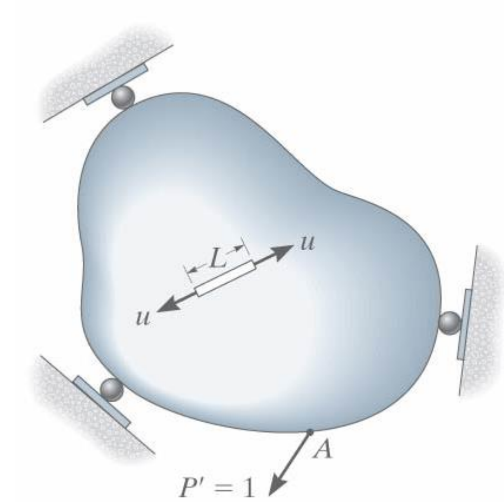
$$\sum P\Delta = \sum u\delta$$

Work of External Loads — Work of Internal Loads

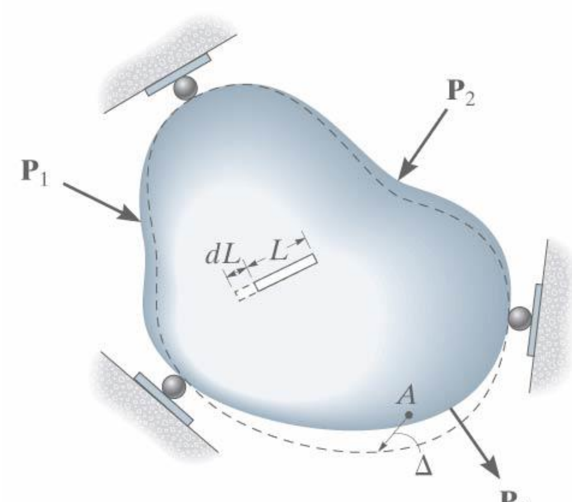
Virtual Load

$$1.\Delta = \sum u.dL$$

Real displacement



Apply virtual load **P′** first.

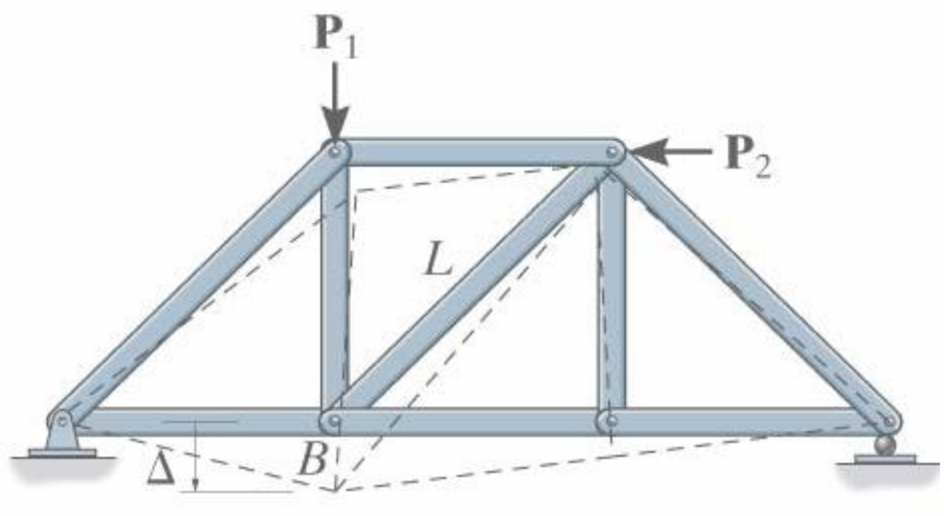


Then apply real loads $\mathbf{P}_1$, $\mathbf{P}_2$, $\mathbf{P}_3$.

Method of Virtual Work: Trusses

$$1 \cdot \Delta = \sum u \cdot dL$$

$$1 \cdot \Delta = \sum n \cdot \frac{NL}{AE}$$



1 = external virtual unit load acting on the truss joint in the stated direction of Δ

n = internal virtual normal force in a truss member caused by the external virtual unit load

Δ = external joint displacement caused by the real load on the truss

N = internal normal force in a truss member caused by the real load

L = length of member

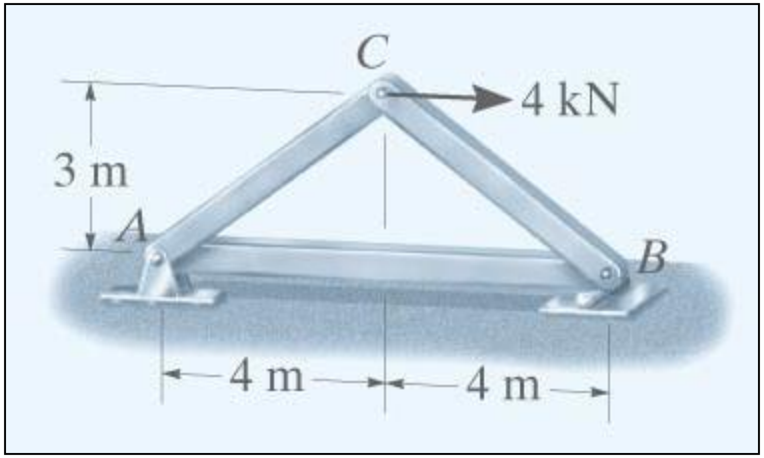
A = cross-sectional area of a member

E = modulus of elasticity of a member

Example 1

The cross sectional area of each member of the truss show, is A = 400mm2 & E = 200GPa.

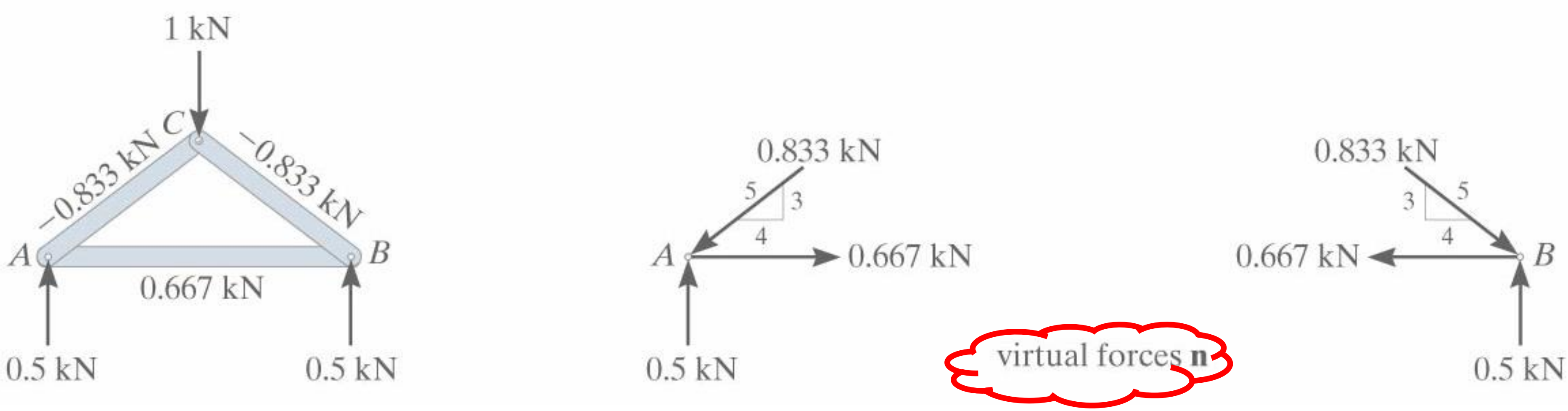
a) Determine the vertical displacement of joint C if a 4-kN force is applied to the truss at C

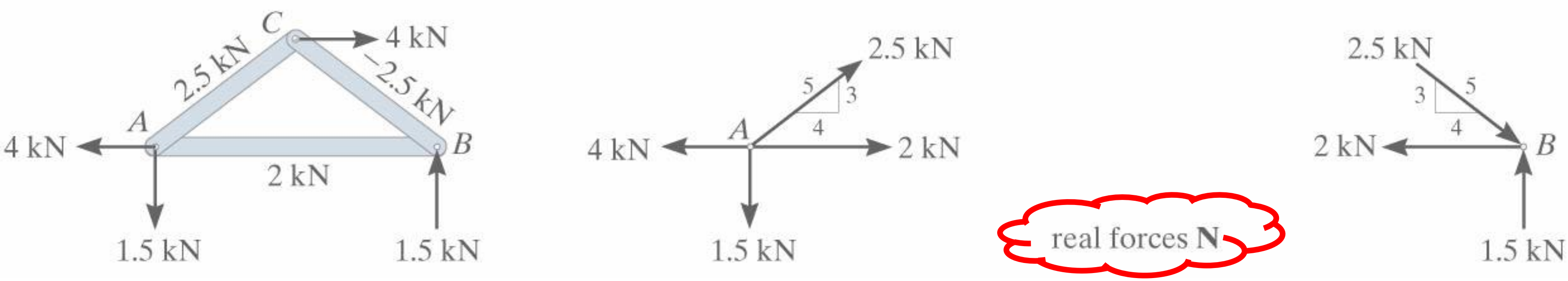


Solution

$$1 . \Delta = \sum n . \frac{NL}{AE}$$

A virtual force of 1 kN is applied at C in the vertical direction





Member	*n (KN)*	*N (KN)*	*L (m)*	*nNL*
AB	**0.667**	**2**	**8**	**10.67**
AC	**-0.833**	**2.5**	**5**	**-10.41**
CB	**-0.833**	**-2.5**	**5**	**10.41**
				Sum 10.67

$$1.\ \Delta = \sum \frac{nNL}{AE} = \frac{10.67}{AE} = \frac{10.67_{(kN)})}{400 \times 10^{-6}{}_{(m^2)} \times 200 \times 10^{6}{}_{(kN/m^2)}}$$

$$\Delta = 0.000133\ m = 0.133\ mm$$

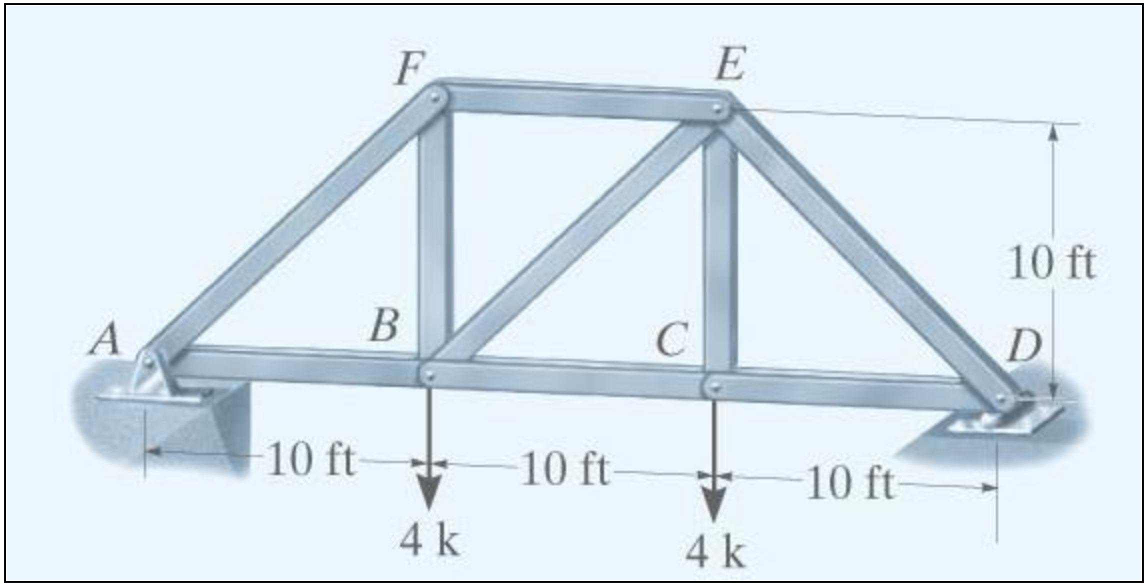
Group work 1

Text book Example 8-14

Determine vertical displacement at C

$A = 0.5\ in^2$

$E = 29\ (10)^3$ ksi



Method of Virtual Work: Beam

$$1.\Delta = \sum u.dL$$

$$d\theta = \frac{M}{EI}dx$$

$$1.\Delta = \int_0^L m \frac{M}{EI}dx$$

1 = external virtual unit load acting on the truss joint in the stated direction of Δ

m = internal virtual moment in a truss member caused by the external virtual unit load

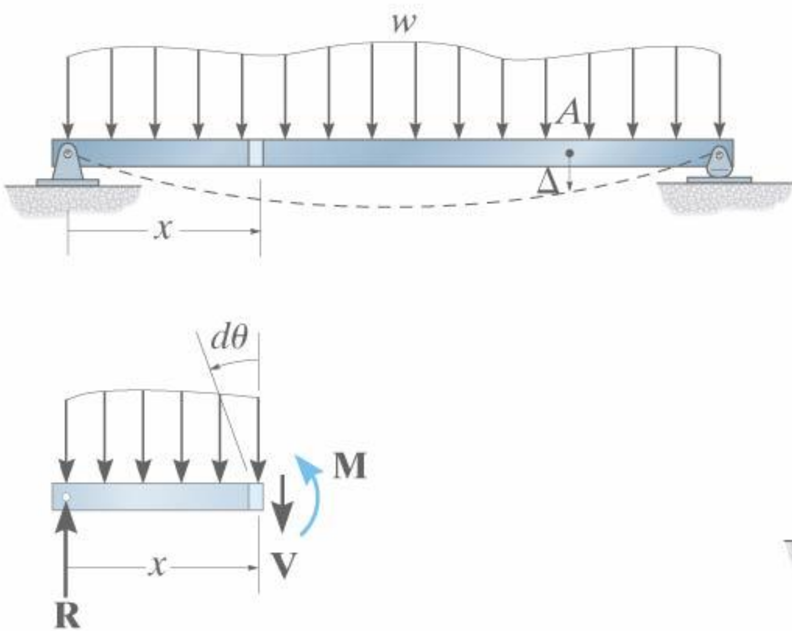
Δ = external joint displacement caused by the real load on the truss

M = internal moment in a beam caused by the real load

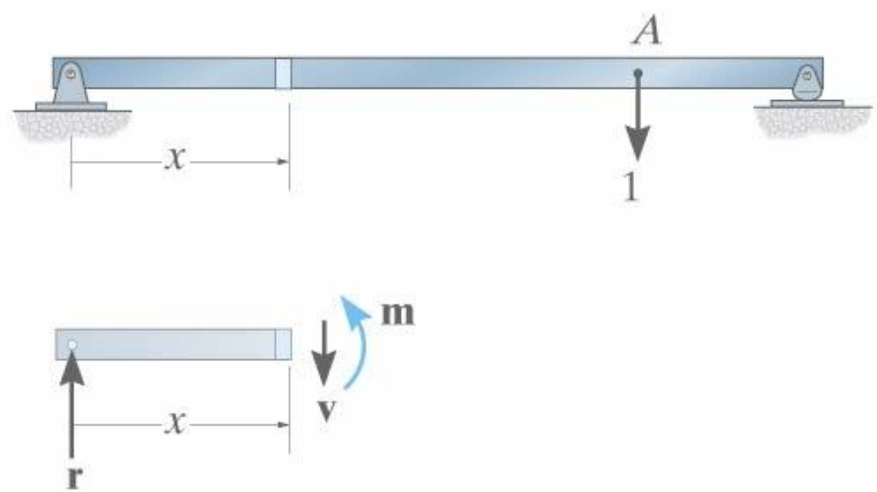
L = length of member

I = moment of inertia of cross-sectional

E = modulus of elasticity of a member



real loads



virtual loads

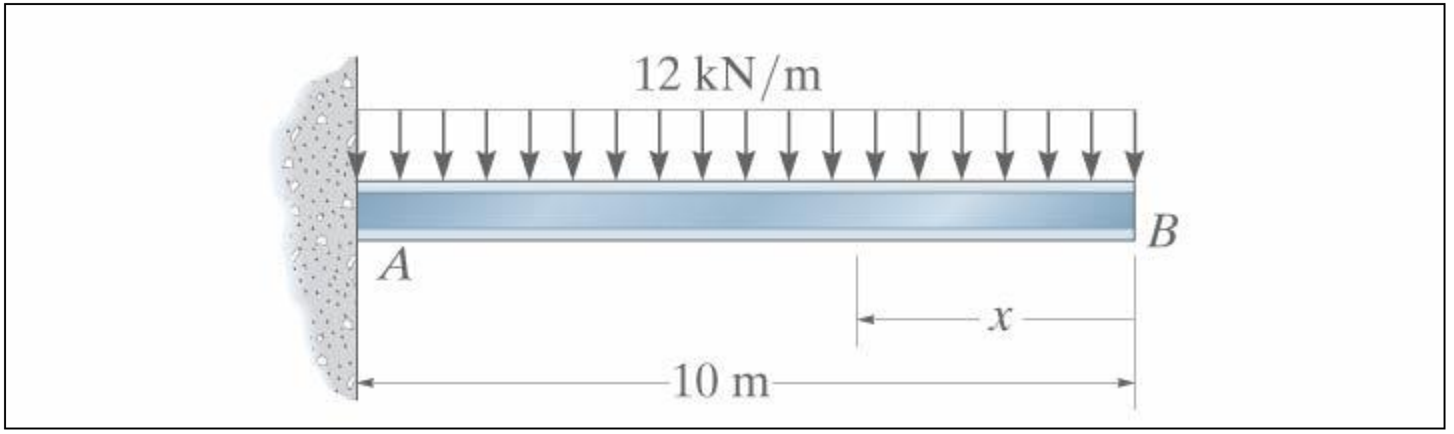
Method of Virtual Work: Beam

Similarly the rotation angle at any point on the beam can be determine, a unit couple moment is applied at the point and the corresponding internal moment m_θ have to be determine

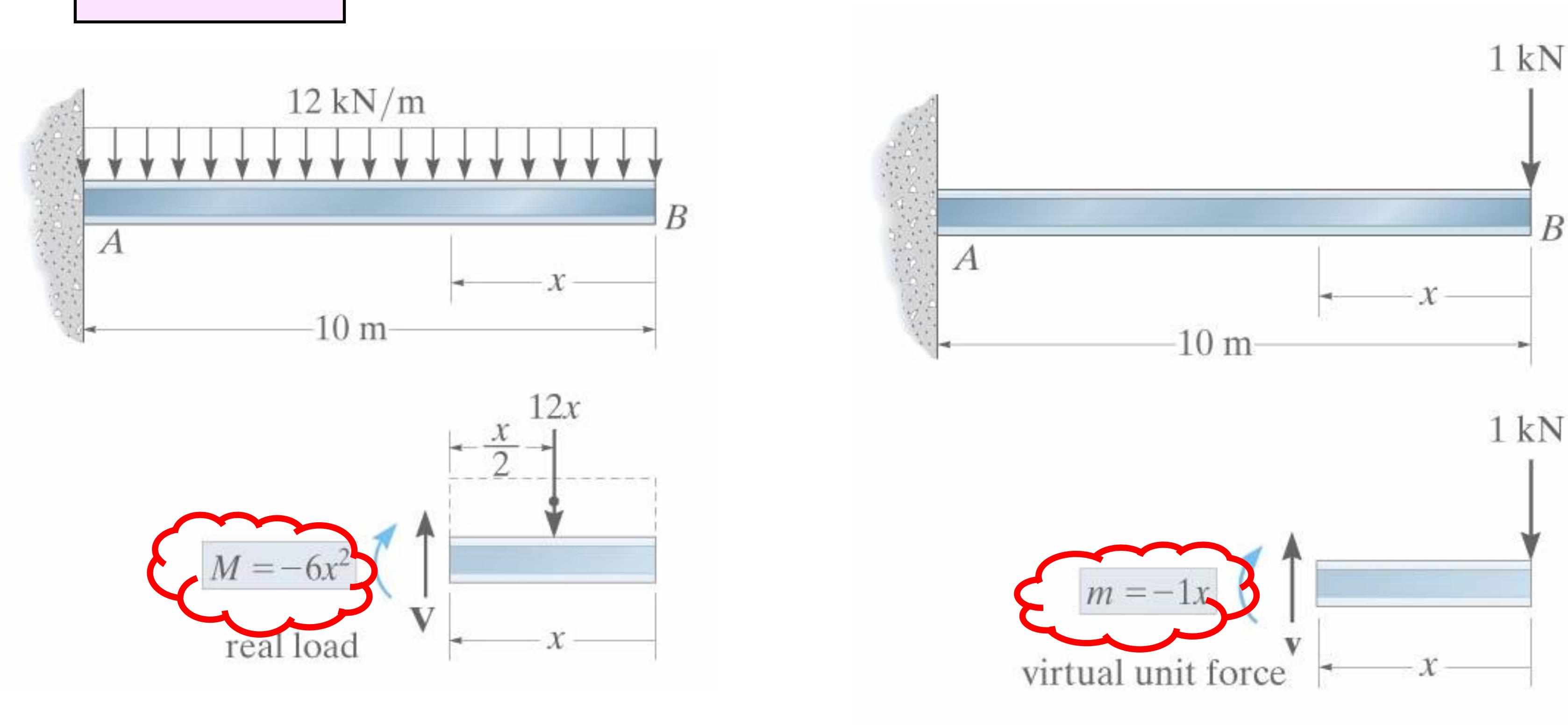
$$1_{(KN.m)}.\theta = \int_0^L \frac{m_\theta M}{EI} dx$$

Example 2

Determine the displacement at point B of a steel beam
E = 200 Gpa , I = 500(10^6) mm^4



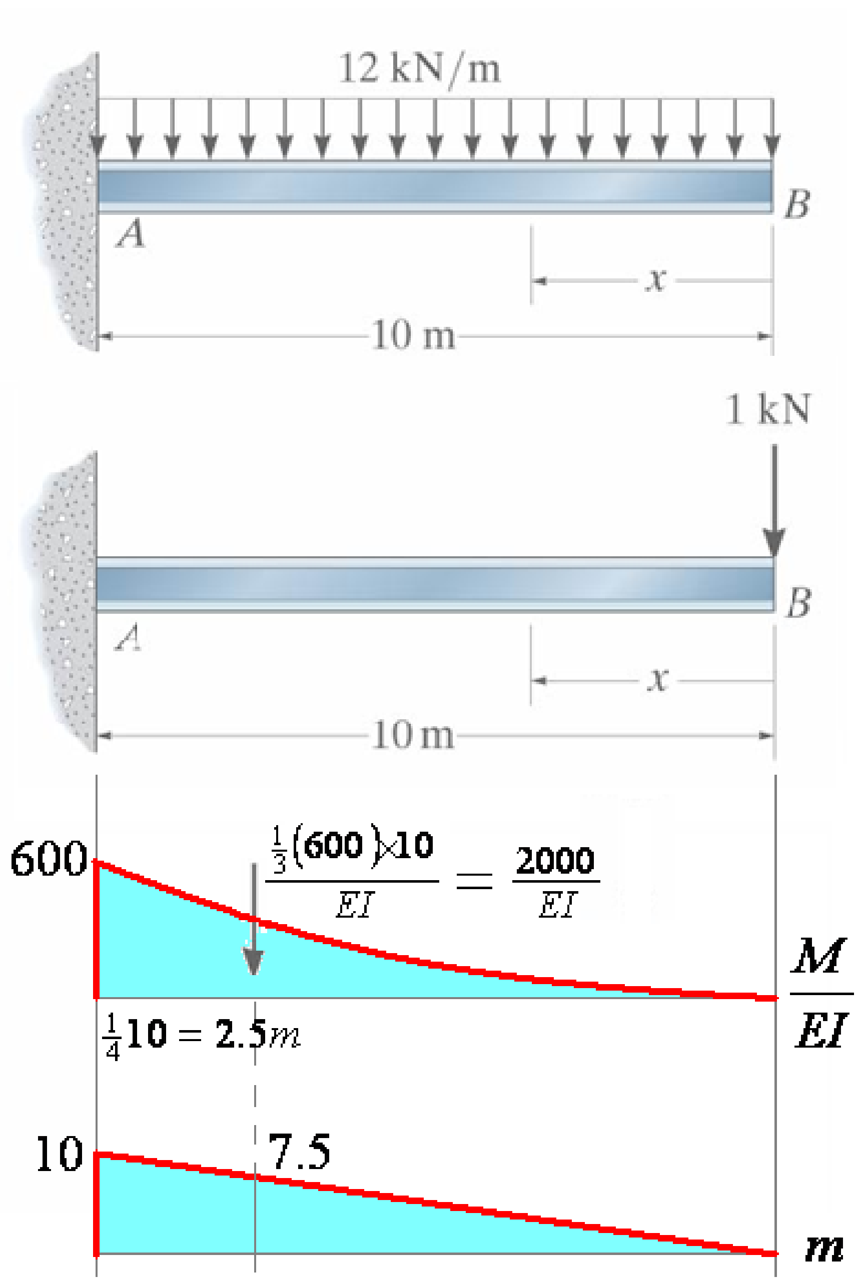
Solution



$$1.\Delta = \int_0^L m\frac{M}{EI}dx = \int_0^{10}\frac{(-1x)\times(-6x^2)dx}{EI} = \int_0^{10}\frac{6x^3dx}{EI} = \left[\frac{6x^4}{4EI}\right]_0^{10}$$

$$\Delta = \frac{15(10^3)}{EI} = \frac{15(10^3)}{200(10^6)\times 500(10^6)(10^{-12})} = 0.15\ m$$

Another Solution



Real Load

Virtual Load

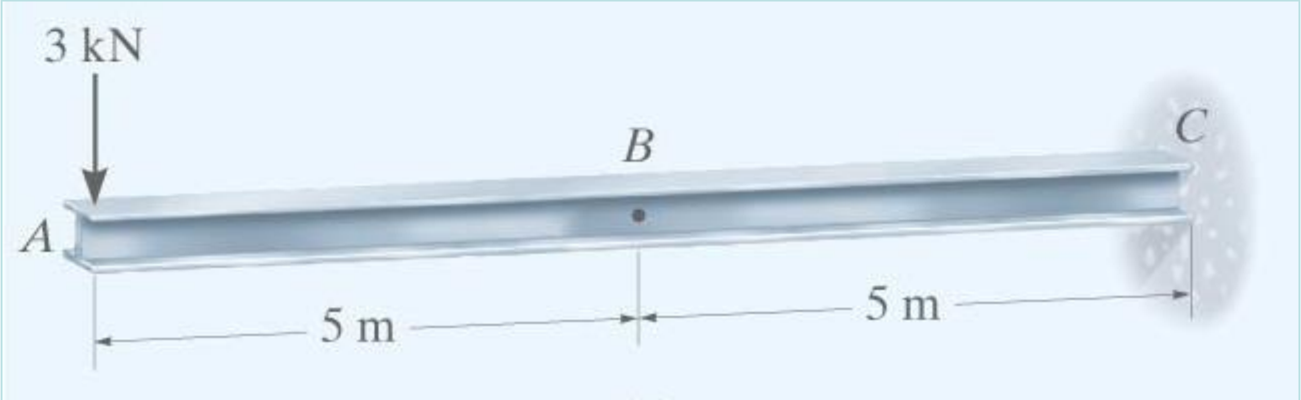
$$\Delta_B = \frac{-2000}{200(10^6)\times 500(10^6)(10^{-12})} \times -7.5$$

$$\Delta_B = 0.15\, m$$

Example 3

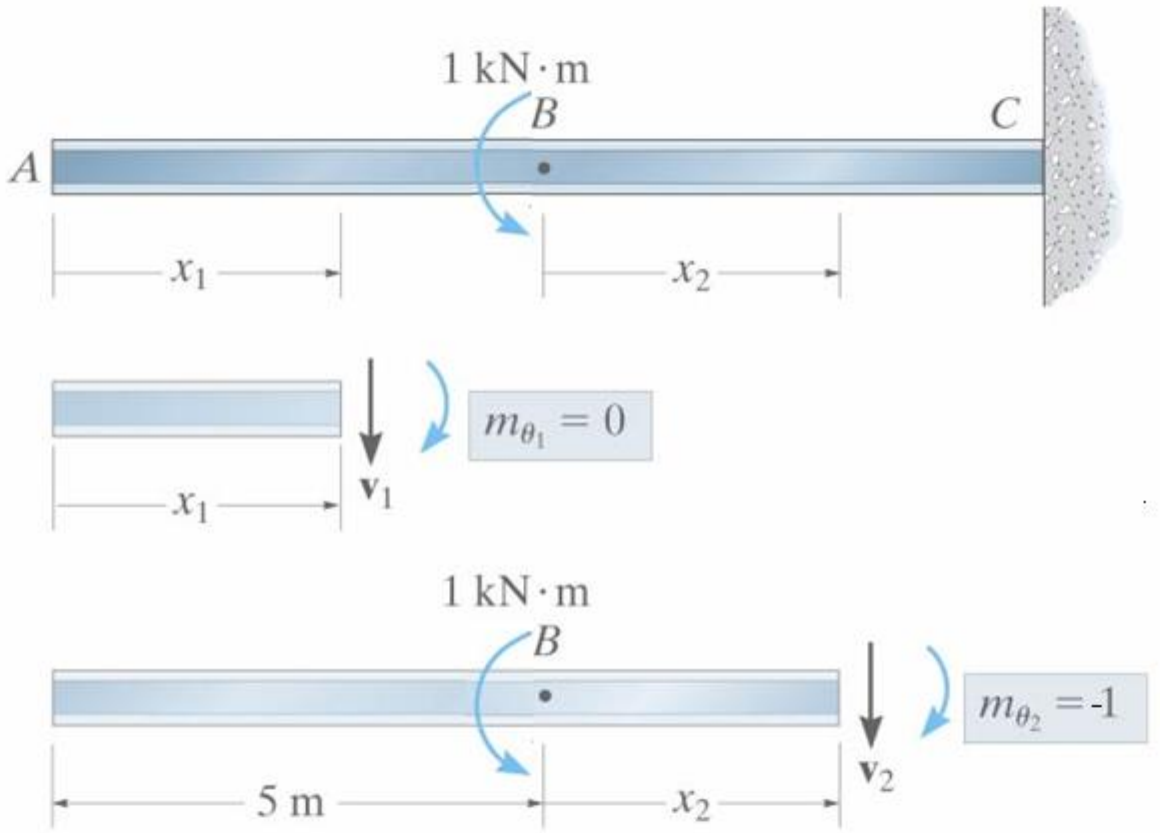
Determine the Slope θ and displacement at point B of a steel beam

E = 200 Gpa , I = $60(10^6)$ mm^4



(a)

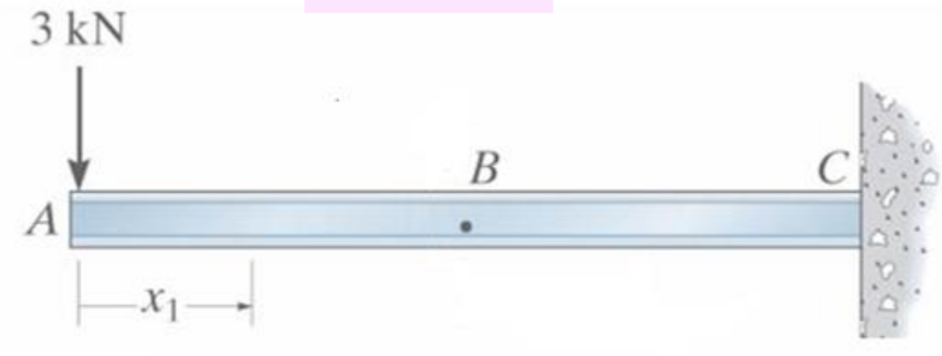
Solution

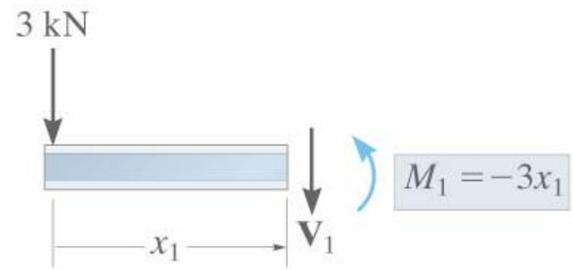


virtual unit couple

Virtual Load

Real Load

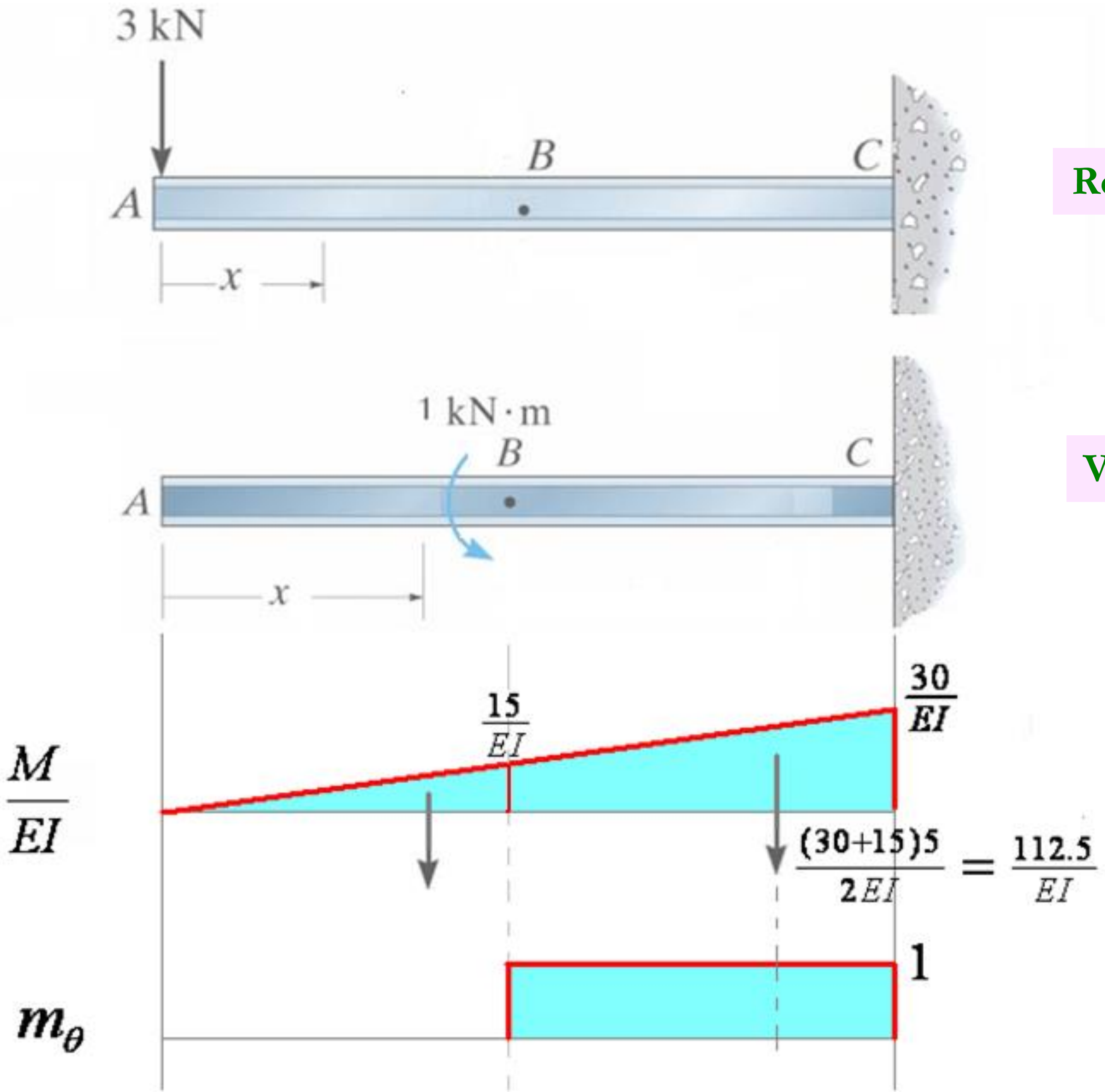




$$1.\theta = \int_0^L m_\theta \frac{M}{EI} dx = \int_0^5 \frac{(0)\times(-3x)\,dx}{EI} + \int_5^{10} \frac{(-1)\times(-3x)\,dx}{EI} = \int_5^{10} \frac{3x\;dx}{EI} = \left[\frac{3x^2}{2EI}\right]_5^{10}$$

$$\theta_B = \frac{3(10^2) - 3(5^2)}{2\times 200\,(10^6)\times 60\,(10^{-6})} = 0.0094 \text{ rad}$$

Another Solution



Real Load

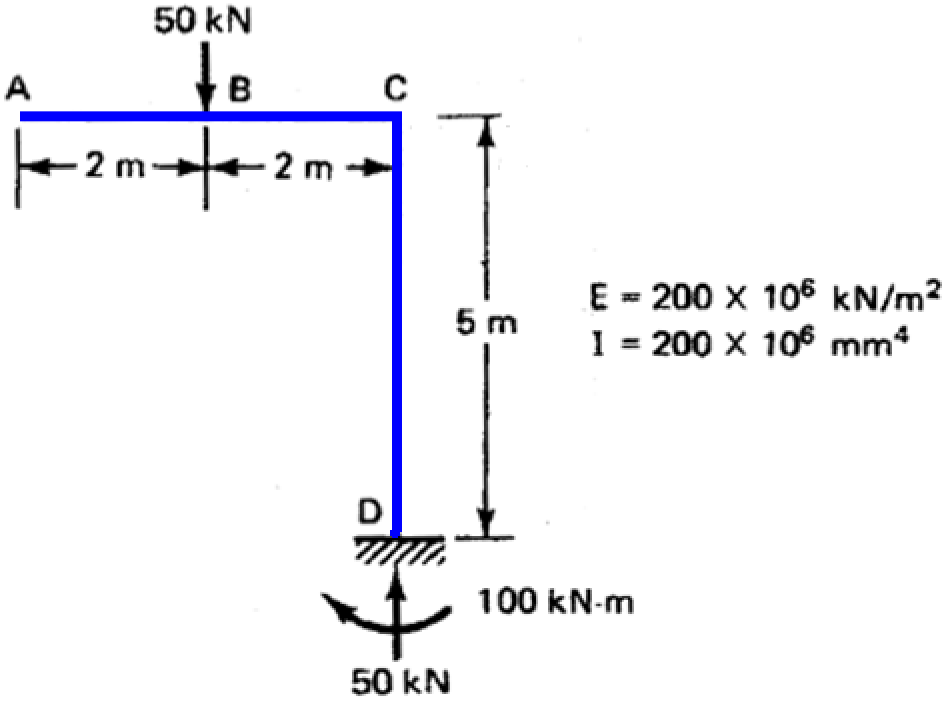
Virtual Load

$$\theta_B = \frac{-112}{EI} \times -1 = \frac{112}{EI}$$

$$= \frac{112}{200\ (10^9) \times 60 \times 10^{-6}} = 0.0094 \quad rad$$

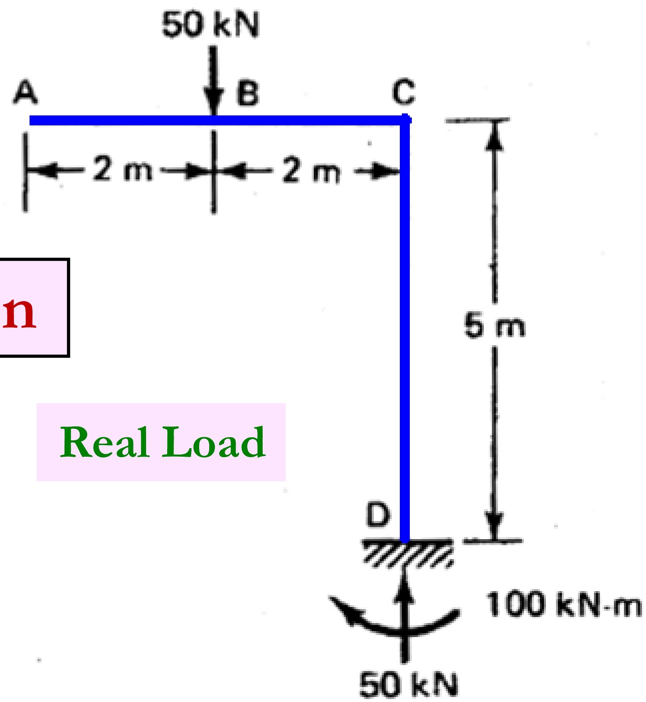
Example 4

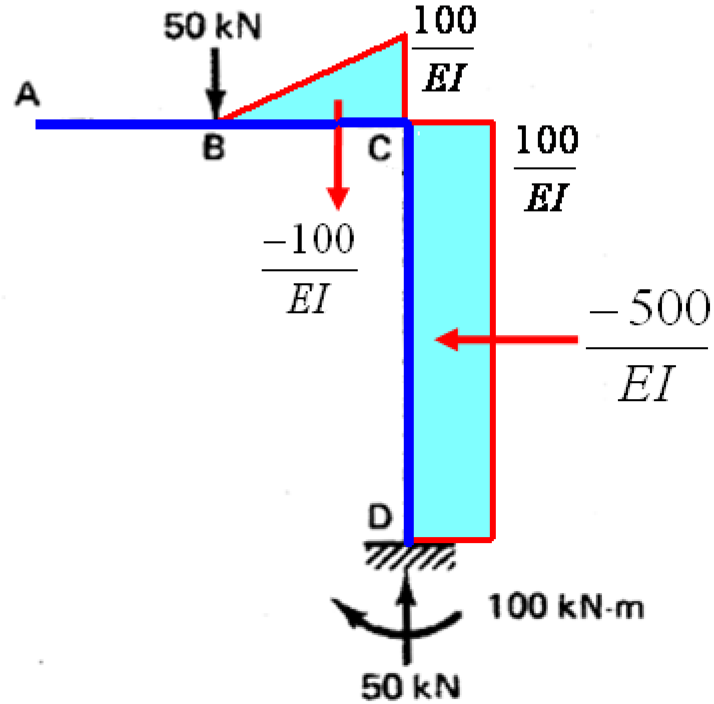
Determine both the horizontal deflection at A



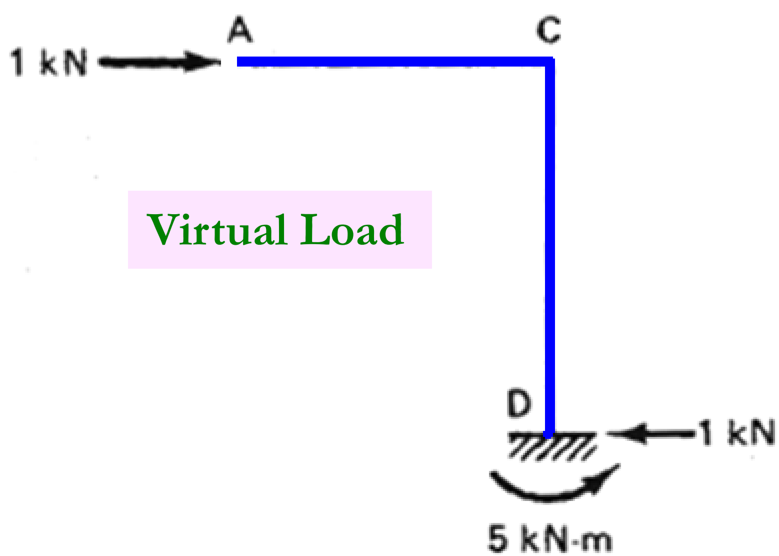
Solution

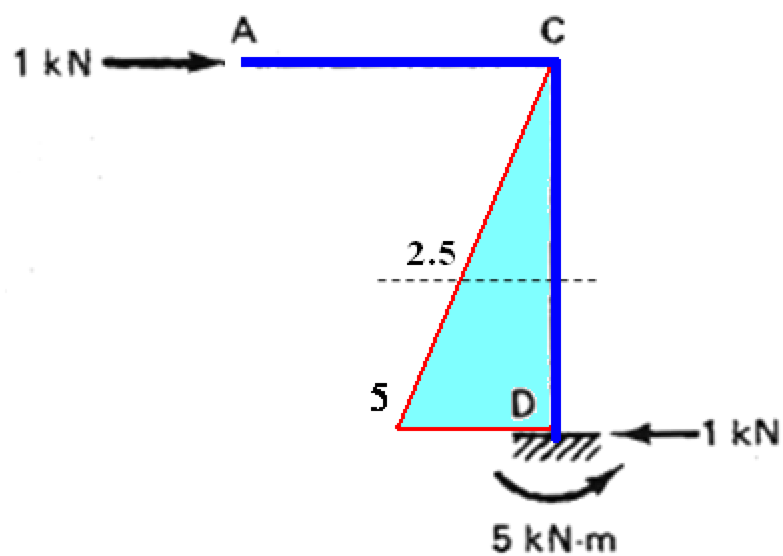
Real Load





Virtual Load





$$\Delta_A = \frac{-100}{EI} \times 0 + \frac{-500}{EI} \times 2.5 = \frac{-1250}{EI}$$

$$= \frac{-1250}{200(10^9) \times 200 \times 10^{-6}} = -0.031\ m$$

Group Work 2

Determine both the Vertical deflection at *C*