ANALYSIS OF PIN JOINTED FRAME

Chapter Objectives

- Determine the forces in the members of a truss using the method of joints and the method of sections
- Analyze forces acting on the members of frames and machines composed of pin-connected members

Chapter Outline

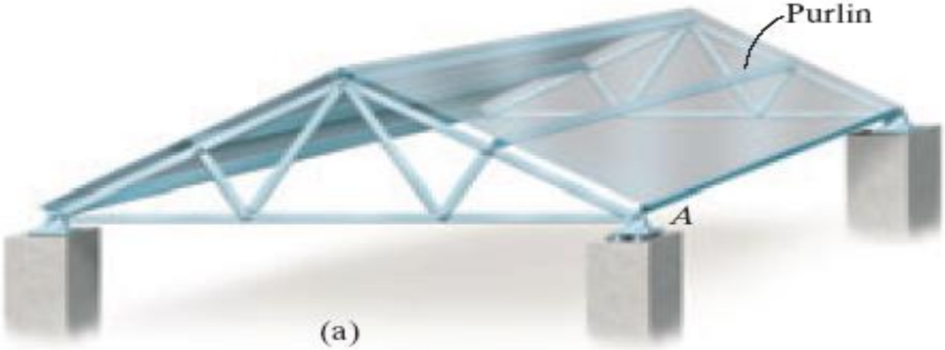
1. Simple Trusses
2. The Method of Joints
3. Zero-Force Members
4. The Method of Sections
5. Space Trusses
6. Frames and Machines

6.1 Simple Trusses

- A truss composed of slender members joined together at their end points

Planar Trusses

- Planar trusses used to support roofs and bridges
- Roof load is transmitted to the truss at joints by means of a series of purlins

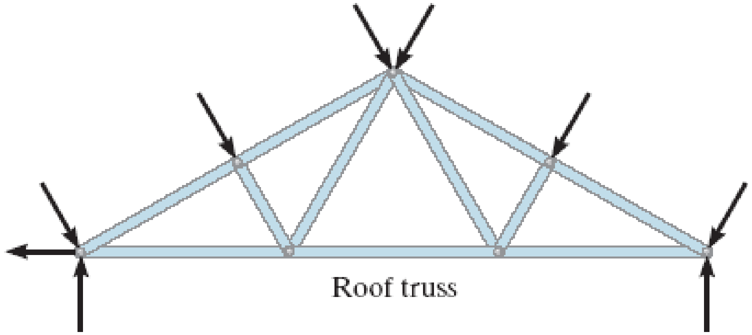


(a)

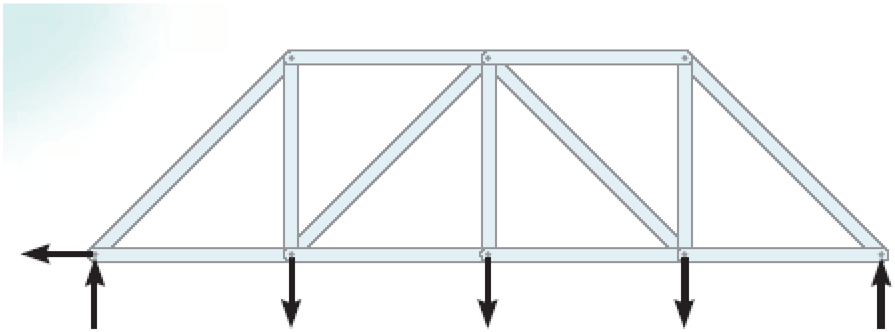
6.1 Simple Trusses

Planar Trusses

- The analysis of the forces developed in the truss members is 2D
- Similar to roof truss, the bridge truss loading is also coplanar



Roof truss

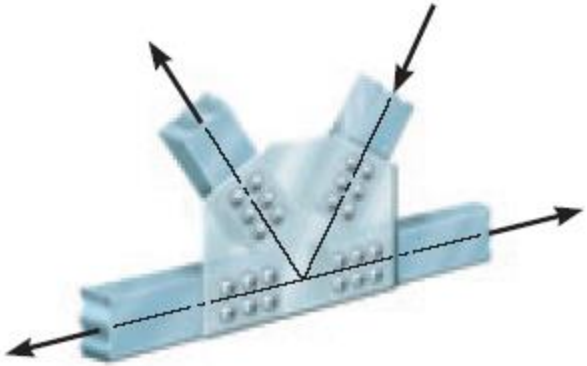


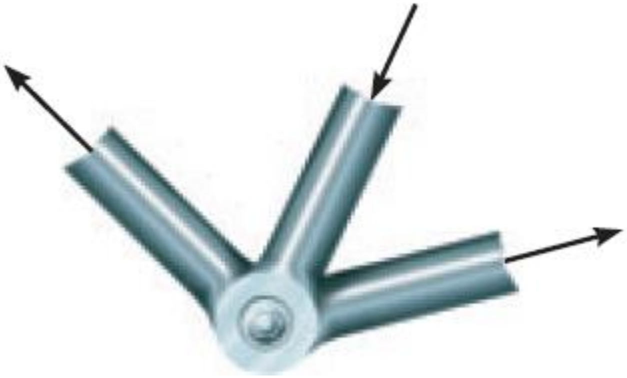
Bridge truss

6.1 Simple Trusses

Assumptions for Design

1. "All loadings are applied at the joint"
   - Weight of the members neglected
2. "The members are joined together by smooth pins"
   - Assume connections provided the center lines of the joining members are *concurrent*

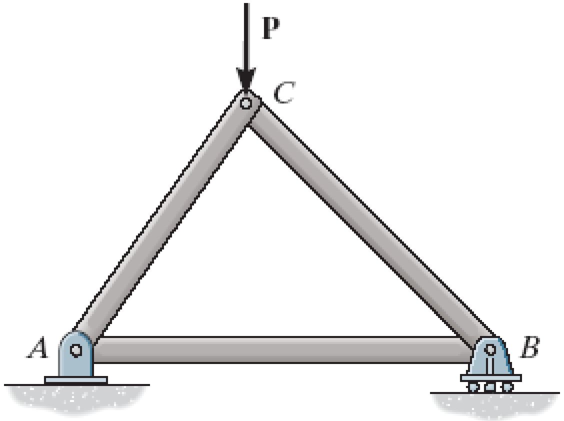


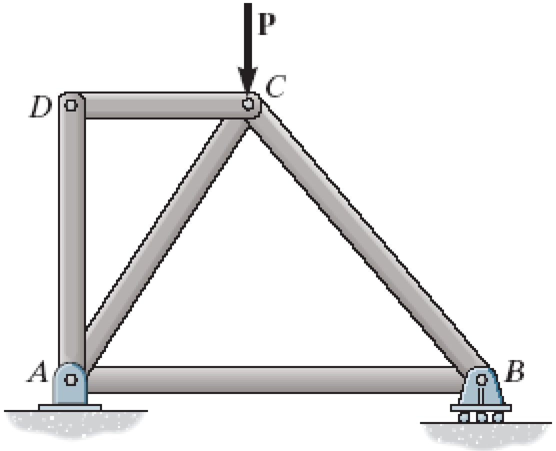


6.1 Simple Trusses

Simple Truss

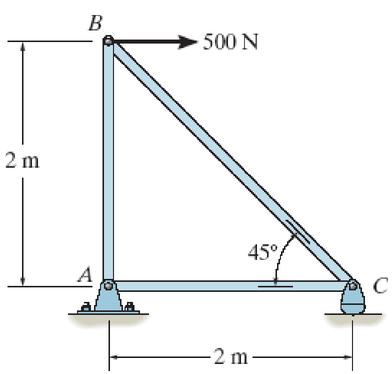
- Form of a truss must be rigid to prevent collapse
- The simplest form that is rigid or stable is a triangle

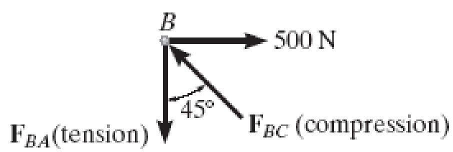


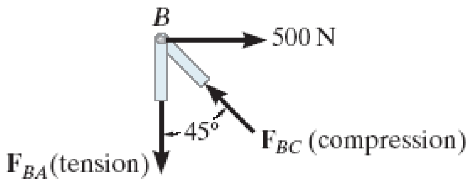


6.2 The Method of Joints

- For truss, we need to know the force in each members
- Forces in the members are internal forces
- For external force members, equations of equilibrium can be applied
- Force system acting at each joint is coplanar and concurrent
- $\sum F_x = 0$ and $\sum F_y = 0$ must be satisfied for equilibrium







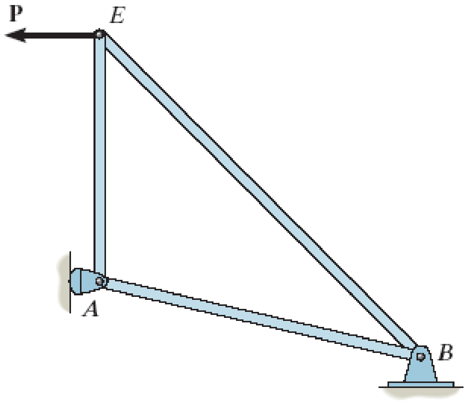
6.2 The Method of Joints

Procedure for Analysis

- Draw the FBD with at least 1 known and 2 unknown forces
- Find the external reactions at the truss support
- Determine the correct sense of the member
- Orient the x and y axes
- Apply $\sum F_x = 0$ and $\sum F_y = 0$
- Use known force to analyze the unknown forces

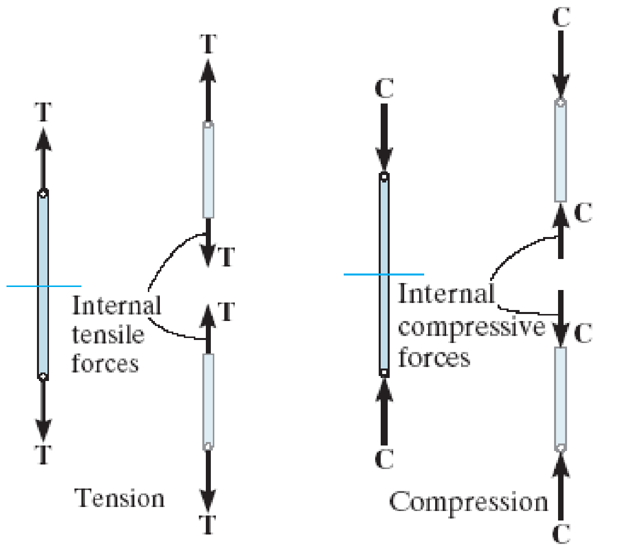
6.3 Zero-Force Members

- Method of joints is simplified using zero-force members
- Zero-force members is supports with no loading
- In general, when 3 members form a truss joint, the 3rd member is a zero-force member provided no external force or support reaction is applied to the joint



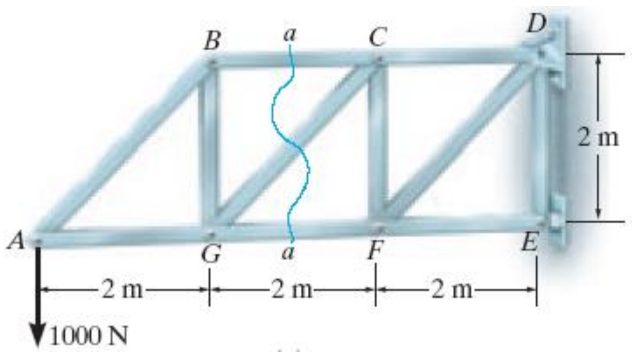
6.4 The Method of Sections

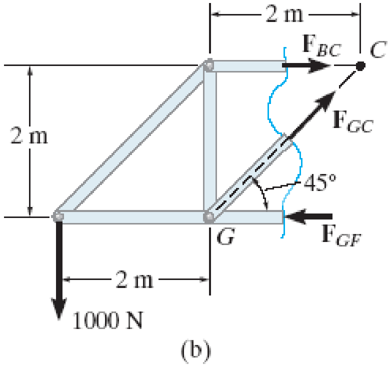
- Used to determine the loadings within a body
- If a body is in equilibrium, any part of the body is in equilibrium
- To find forces within members, an imaginary section is used to cut each member into 2 and expose each internal force as external

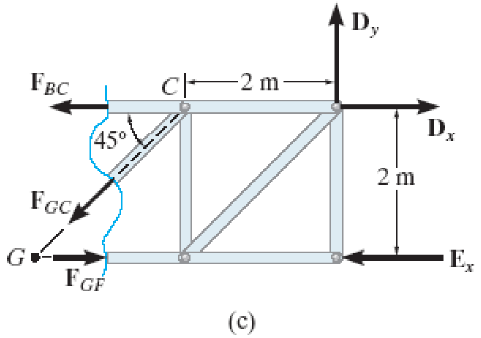


6.4 The Method of Sections

- Consider the truss and section a-a as shown
- Member forces are equal and opposite to those acting on the other part – Newton's Law







6.4 The Method of Sections

Procedure for Analysis

Free-Body Diagram

- Decide the section of the truss
- Determine the truss's external reactions
- Use equilibrium equations to solve member forces at the cut session
- Draw FBD of the sectioned truss which has the least number of forces acting on it
- Find the sense of an unknown member force

6.4 The Method of Sections

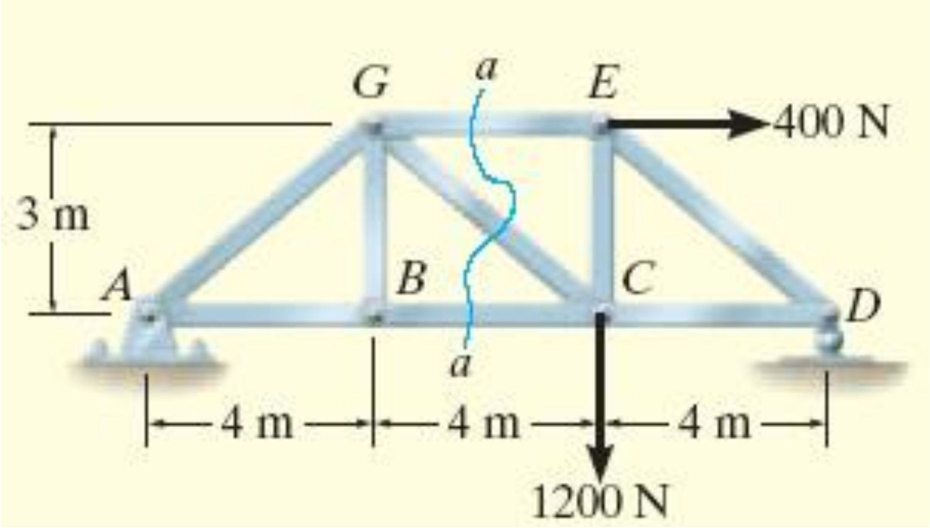
Procedure for Analysis

Equations of Equilibrium

- Summed moments about a point
- Find the 3rd unknown force from moment equation

Example 6.5

Determine the force in members GE, GC, and BC of the truss. Indicate whether the members are in tension or compression.



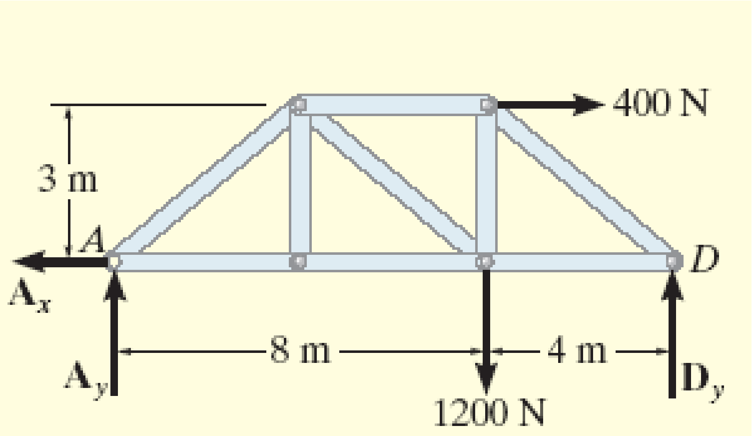
Solution

- Choose section a-a since it cuts through the three members
- Draw FBD of the entire truss

$$+\rightarrow \sum F_x = 0; \qquad 400\ N - A_x = 0 \Rightarrow A_x = 400\ N$$

$$\sum M_A = 0; \qquad -1200\ N(8m) - 400\ N(3m) + D_y(12\ m) = 0 \Rightarrow D_y = 900\ N$$

$$+\uparrow \sum F_y = 0; \qquad A_y - 1200\ N + 900\ N = 0 \Rightarrow A_y = 300\ N$$



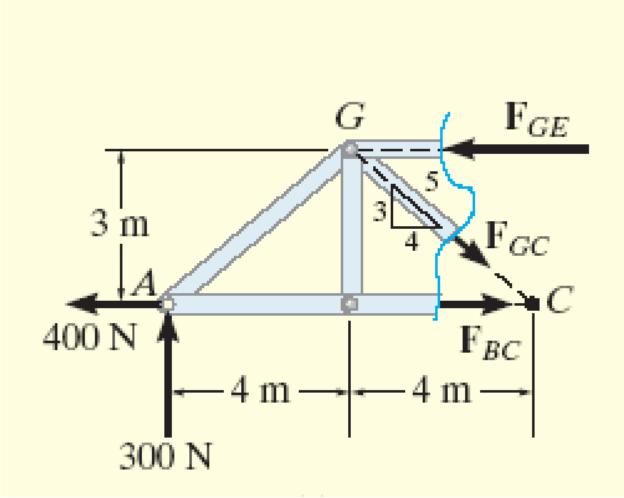
Solution

- Draw FBD for the section portion

$$\sum M_G = 0; \qquad -300\ N(4m) - 400\ N(3m) + F_{BC}(3m) = 0 \Rightarrow F_{BC} = 800\ N\ (T)$$

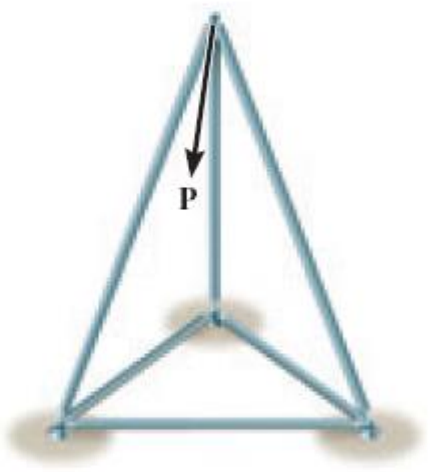
$$\sum M_C = 0; \qquad -300\ N(8m) + F_{GE}(3m) = 0 \Rightarrow F_{GE} = 800\ N\ (C)$$

$$+\uparrow \sum F_y = 0; \qquad 300\ N - \frac{3}{5} F_{GC} = 0 \Rightarrow F_{GC} = 500\ N\ (T)$$



6.5 Space Trusses

- Consists of members joined together at their ends to form 3D structure
- The simplest space truss is a tetrahedron
- Additional members would be redundant in supporting force **P**



6.5 Space Trusses

Assumptions for Design

- Members of a space truss is treated as 2 force members provided the external loading is at the joints
- When weight of the member is considered, apply it as a vertical force, half of its magnitude applied at each end of the member

Method of Joints

- Solve $\sum F_x = 0$, $\sum F_y = 0$, $\sum F_z = 0$ at each joint
- Force analysis has at least 1 unknown force and 3 unknown forces

6.5 Space Trusses

Method of Sections

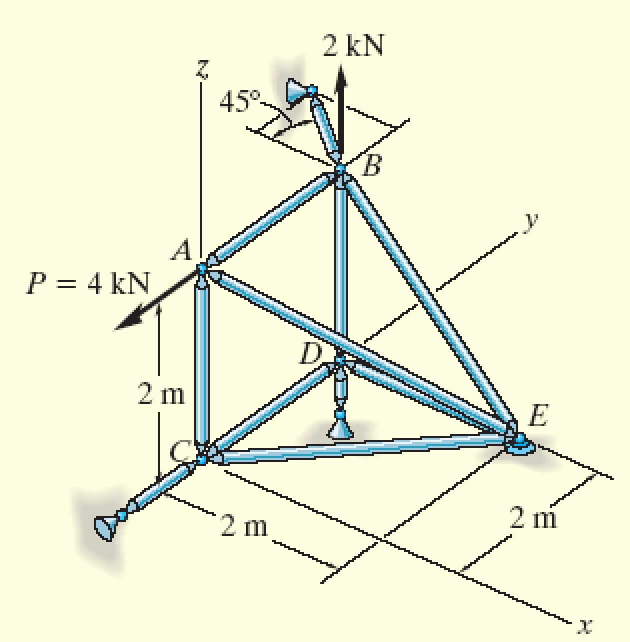
- When imaginary section is passes through a truss it must satisfied

$$\sum F_x = 0, \sum F_y = 0, \sum F_z = 0$$
$$\sum M_x = 0, \sum M_y = 0, \sum M_z = 0$$

- By proper selection, the unknown forces can be determined using a single equilibrium equation

Example 6.8

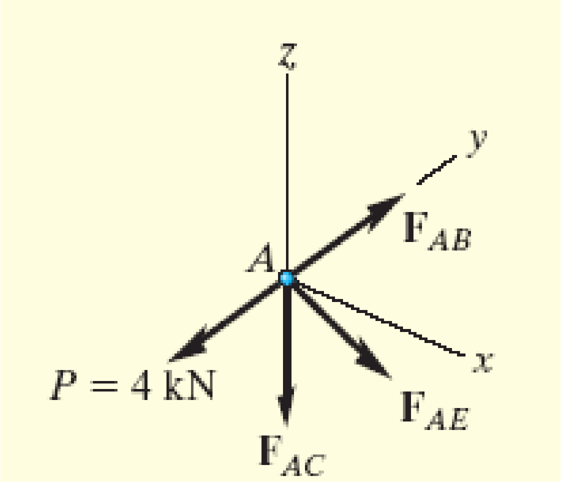
Determine the forces acting in the members of the space truss. Indicate whether the members are in tension or compression.



Solution

For Joint A,

$$\vec{P} = \{-4\vec{j}\}kN, \vec{F}_{AB} = F_{AB}\vec{j}, \vec{F}_{AC} = -F_{AC}\vec{k}$$



$$\vec{F}_{AE} = F_{AE}\left(\frac{\vec{r}_{AE}}{r_{AE}}\right)$$

$$= F_{AE}(0.577\vec{i} + 0.577\vec{j} - 0.577\vec{k})$$

$$\sum \vec{F} = 0;$$

$$\vec{P} + \vec{F}_{AB} + \vec{F}_{AC} + \vec{F}_{AE} = 0$$

$$-4\vec{j} + F_{AB}\vec{j} - F_{AC}\vec{k} + 0.577F_{AE}\vec{i} + 0.577F_{AE}\vec{j} - 0.577F_{AE}\vec{k} = 0$$

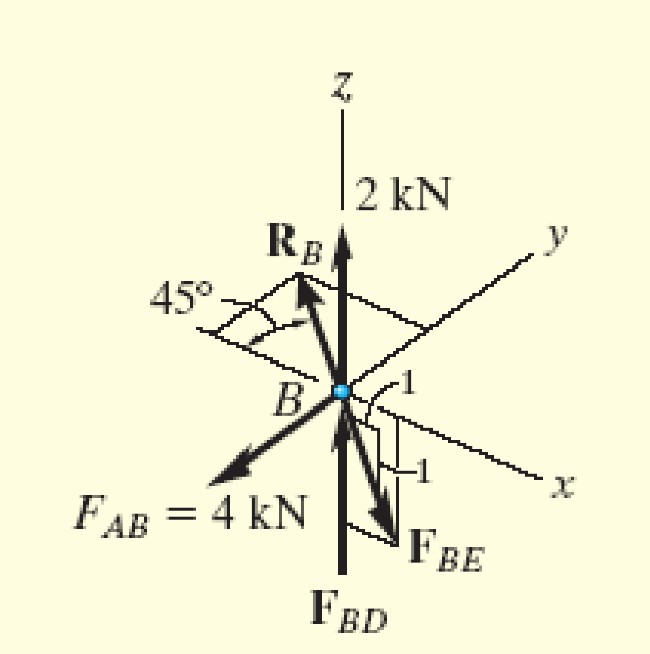
Solution

For Joint B,

$$\sum F_x = 0; -R_B \cos 45^\circ + 0.707 F_{BE} = 0$$

$$\sum F_y = 0; -4 + R_B \sin 45^\circ = 0$$

$$\sum F_z = 0; 2 + F_{BD} - 0.707 F_{BE} = 0$$



$$R_B = F_{BE} = 5.66\ kN\ (T)$$

$$F_{BD} = 2\ kN\ (C)$$

To show,

$$F_{DE} = F_{DC} = F_{CE} = 0$$

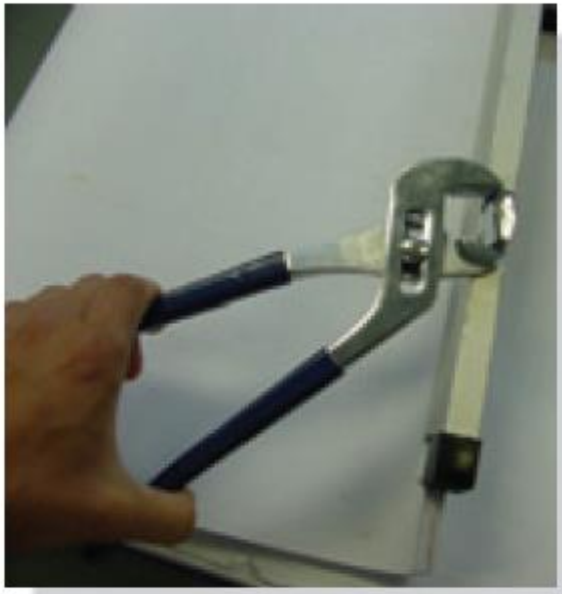
6.6 Frames and Machines

- Composed of pin-connected multi-force members
- Frames are stationary
- Apply equations of equilibrium to each member to determine the unknown forces

6.6 Frames and Machines

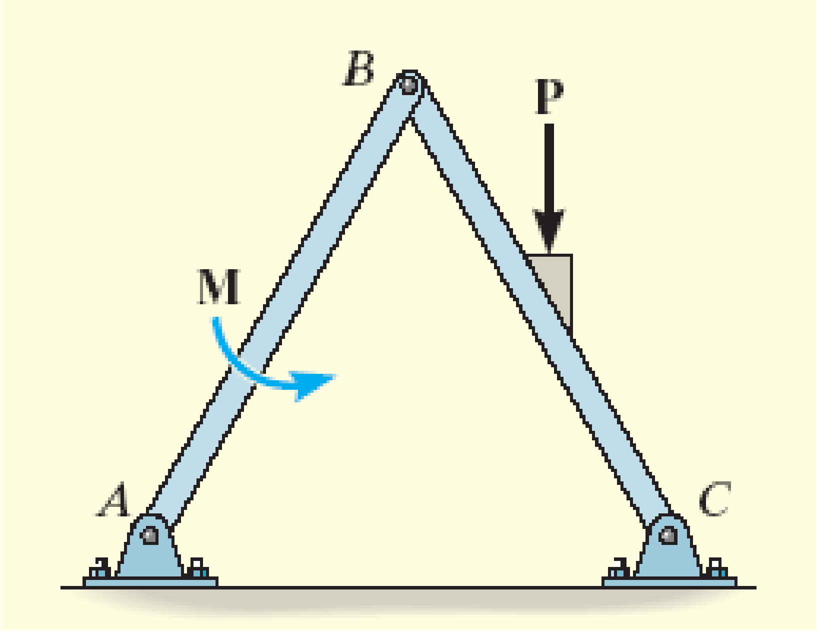
Free-Body Diagram

- Isolate each part by drawing its outlined shape
  - show all forces and couple moments act on the part
  - identify each known and unknown force and couple moment
  - indicate any dimension
  - apply equations of equilibrium
  - assumed sense of unknown force or moment
  - draw FBD



Example 6.9

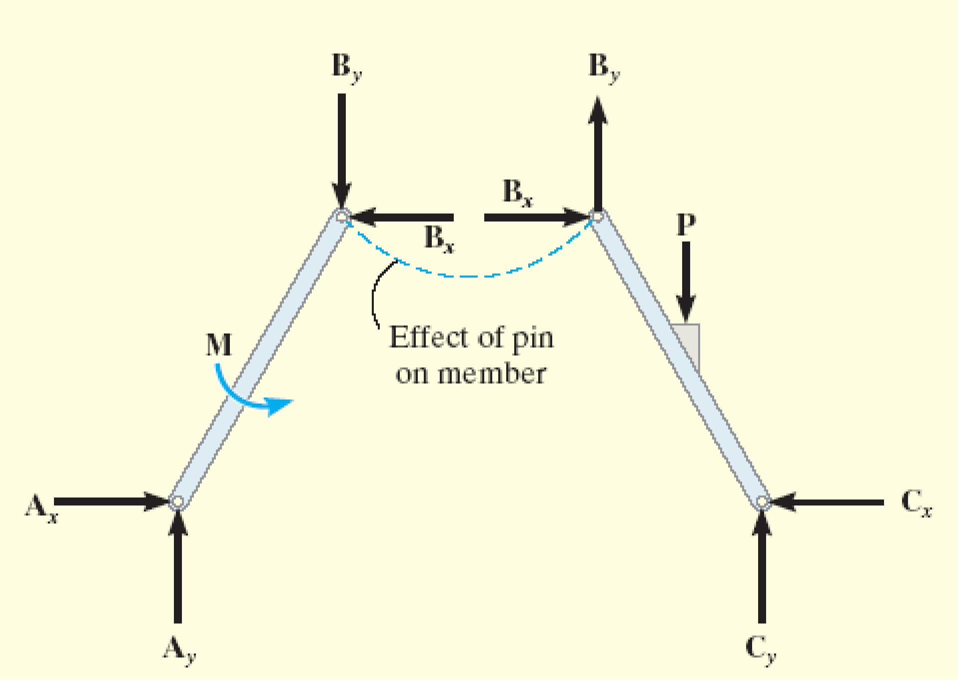
For the frame, draw the free-body diagram of (a) each member, (b) the pin at B and (c) the two members connected together.



Solution

Part (a)

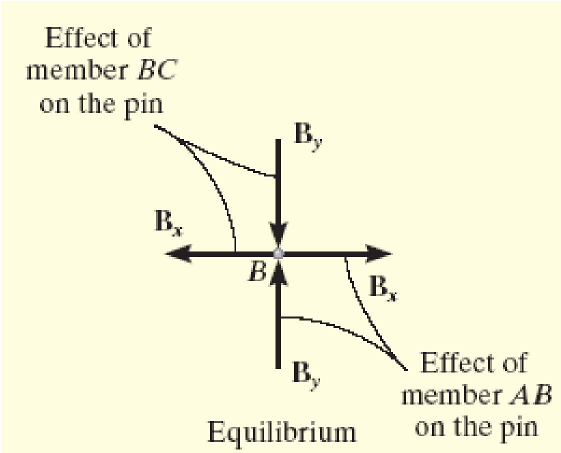
- BA and BC are not two-force
- AB is subjected to the resultant forces from the pins



Solution

Part (b)

- Pin at B is subjected to two forces, force of the member BC and AB on the pin
- For equilibrium, forces and respective components must be equal but opposite
- **B**$_x$ and **B**$_y$ shown equal and opposite on members AB



Solution

Part (c)

- **B**$_x$ and **B**$_y$ are not shown as they form equal but opposite internal forces
- Unknown force at A and C must act in the same sense
- Couple moment **M** is used to find reactions at A and C