# 7. MOMENT DISTRIBUTION METHOD

# 7.1 MOMENT DISTRIBUTION METHOD - AN OVERVIEW

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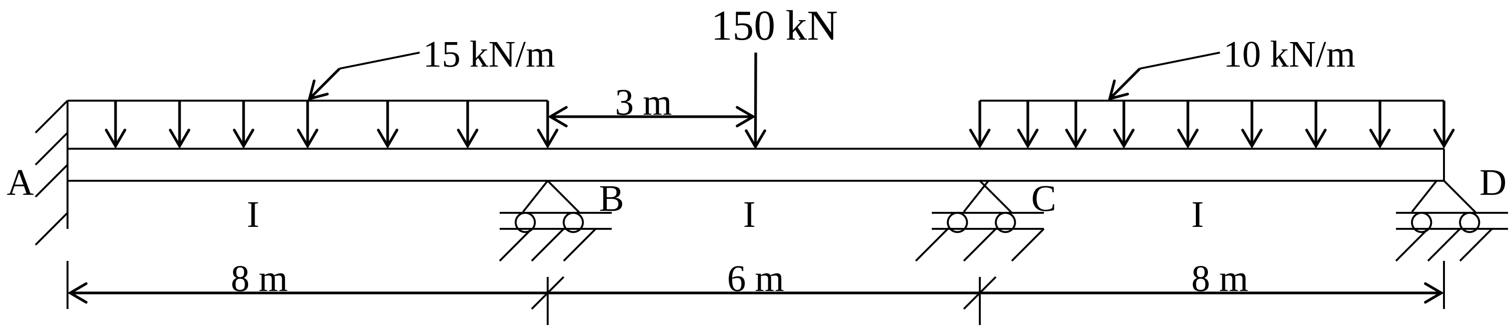
# 7.2 MOMENT DISTRIBUTION METHOD - INTRODUCTION AND BASIC PRINCIPLES

## 7.1 Introduction

(Method developed by Prof. Hardy Cross in 1932)
The method solves for the joint moments in continuous beams and rigid frames by successive approximation.

## 7.2 Statement of Basic Principles

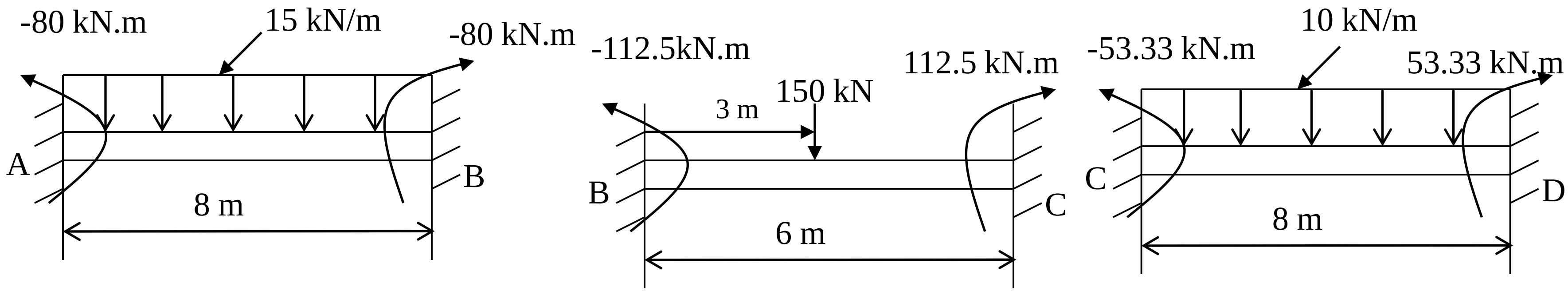
Consider the continuous beam ABCD, subjected to the given loads, as shown in Figure below. Assume that only rotation of joints occur at B, C and D, and that no support displacements occur at B, C and D. Due to the applied loads in spans AB, BC and CD, rotations occur at B, C and D.



In order to solve the problem in a successively approximating manner, it can be visualized to be made up of a continued two-stage problems viz., that of locking and releasing the joints in a continuous sequence.

## 7.2.1 Step I

**The joints B, C and D are locked in position before any load is applied on the beam ABCD; then given loads are applied on the beam. Since the joints of beam ABCD are locked in position, beams AB, BC and CD acts as individual and separate fixed beams, subjected to the applied loads; these loads develop fixed end moments.**



In beam AB

Fixed end moment at A = $-wl^2/12$ = - (15)(8)(8)/12 = - 80 kN.m

Fixed end moment at B = $+wl^2/12$ = +(15)(8)(8)/12 = + 80 kN.m

In beam BC

Fixed end moment at B = $-(Pab^2)/l^2$ = $-(150)(3)(3)^2/6^2$

= -112.5 kN.m

Fixed end moment at C = $+(Pab^2)/l^2$ = $+(150)(3)(3)^2/6^2$

= + 112.5 kN.m

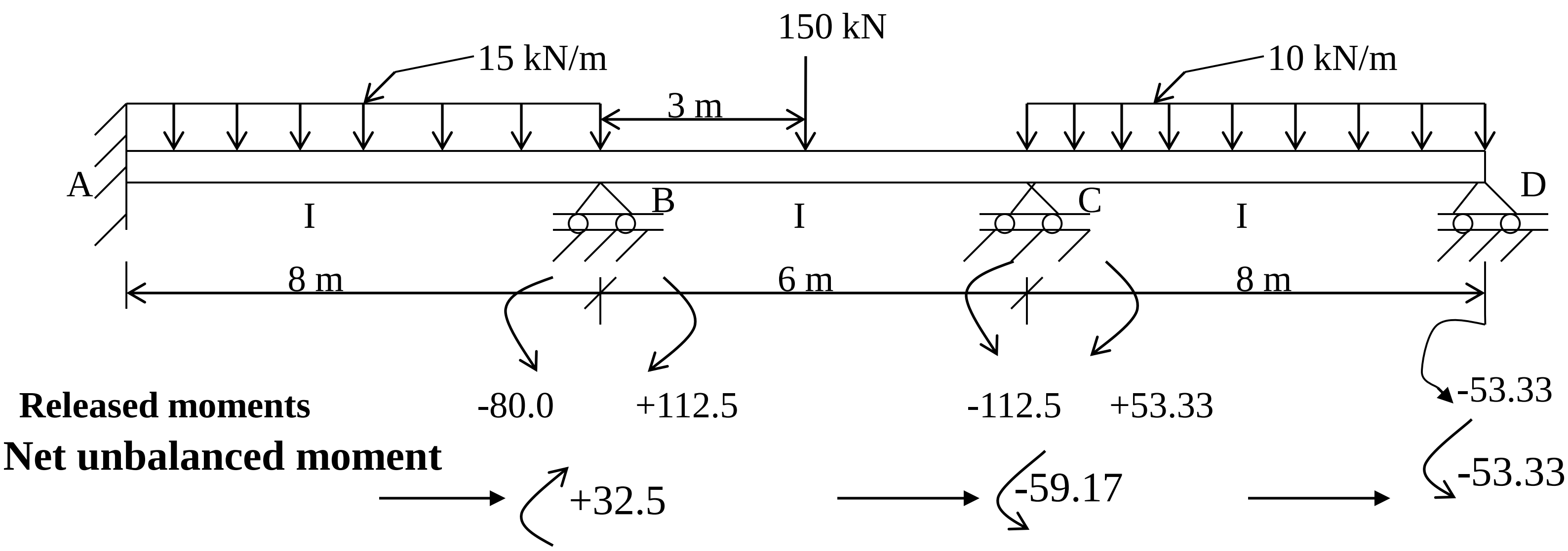
In beam AB

Fixed end moment at C = $-wl^2/12$ = - (10)(8)(8)/12 = - 53.33 kN.m

Fixed end moment at D = $+wl^2/12$ = +(10)(8)(8)/12 = + 53.33kN.m

## 7.2.2 Step II

Since the joints B, C and D were fixed artificially (to compute the the fixed-end moments), now the joints B, C and D are released and allowed to rotate. Due to the  joint release, the joints rotate maintaining the continuous nature of the beam. Due to the joint release, the fixed end moments on either side of joints B, C and D act in the opposite direction now, and cause a net unbalanced moment to occur at the joint.



## 7.2.3 Step III

These unbalanced moments act at the joints and modify the joint moments at B, C and D, according to their relative stiffnesses at the respective joints. The joint moments are distributed to either side of the joint B, C or D, according to their relative stiffnesses. These distributed moments also modify the moments at the opposite side of the beam span, viz., at joint A in span AB, at joints B and C in span BC and at joints C and D in span CD. This modification is dependent on the carry-over factor (which is equal to 0.5 in this case); **when this carry over is made, the joints on opposite side are assumed to be fixed.**

## 7.2.4 Step IV

**The carry-over moment becomes the unbalanced moment at the joints to which they are carried over. Steps 3 and 4 are repeated till the carry-over or distributed moment becomes small**.

## 7.2.5 Step V

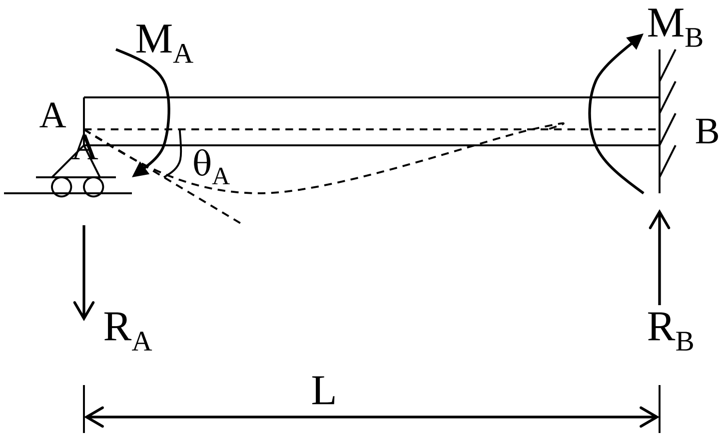
**Sum up all the moments at each of the joint** to obtain the joint moments.

# 7.3 SOME BASIC DEFINITIONS

In order to understand the five steps mentioned in section 7.3, some words need to be defined and relevant derivations made.

## 7.3.1 Stiffness and Carry-over Factors

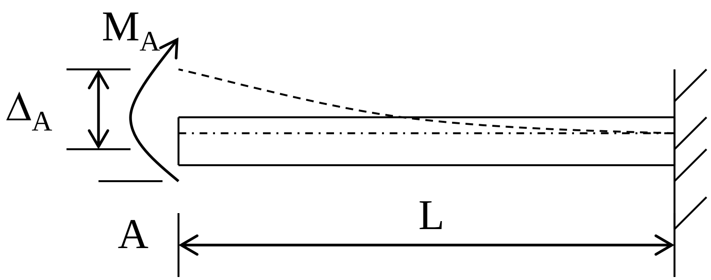
Stiffness = Resistance offered by member to a unit displacement or rotation at a point, for given support constraint conditions



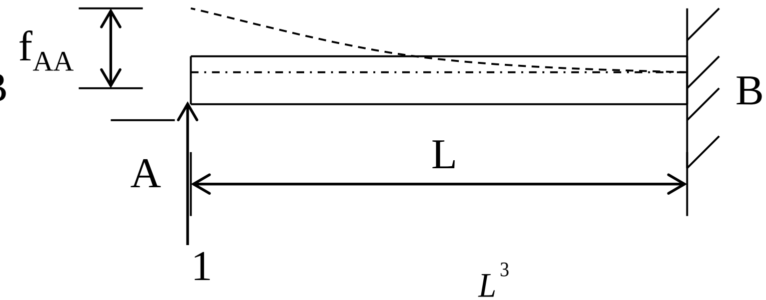

**E, I – Member properties**

A clockwise moment $M_A$ is applied at A to produce a +ve bending in beam AB. Find $\theta_A$ and $M_B$.

# Using method of consistent deformations




$$\Delta_A = +\frac{M_A L^2}{2EI}$$




$$f_{AA} = \frac{L^3}{3EI}$$

**Applying the principle of consistent deformation,**

$$\Delta_{\mathbf{A}} + \mathbf{R}_{\mathbf{A}} \mathbf{f}_{\mathbf{AA}} = 0 \quad \rightarrow \mathbf{R}_{\mathbf{A}} = -\frac{3\mathbf{M}_{\mathbf{A}}}{2\mathbf{L}} \downarrow$$

$$\theta_A = \frac{M_A L}{EI} + \frac{R_A L^2}{2EI} = \frac{M_A L}{4EI} \qquad \therefore\ M_A = \frac{4EI}{L}\theta_A; \quad hence \quad k_\theta = \frac{M_A}{\theta_A} = \frac{4EI}{L}$$

**Stiffness factor = $k_\theta$ = 4EI/L**

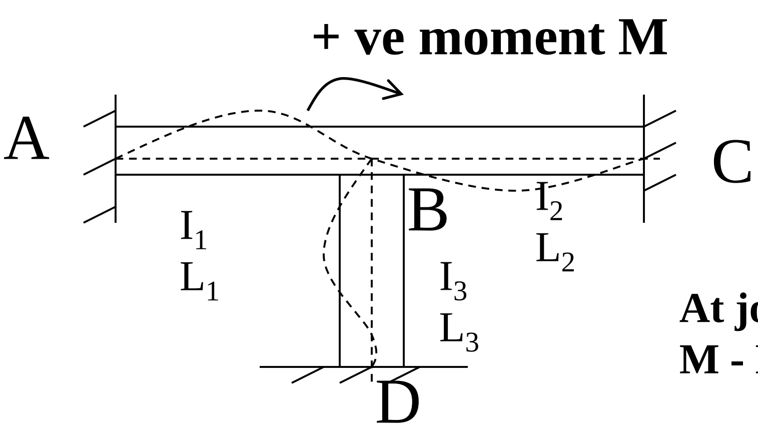
## Considering moment $M_{B,}$

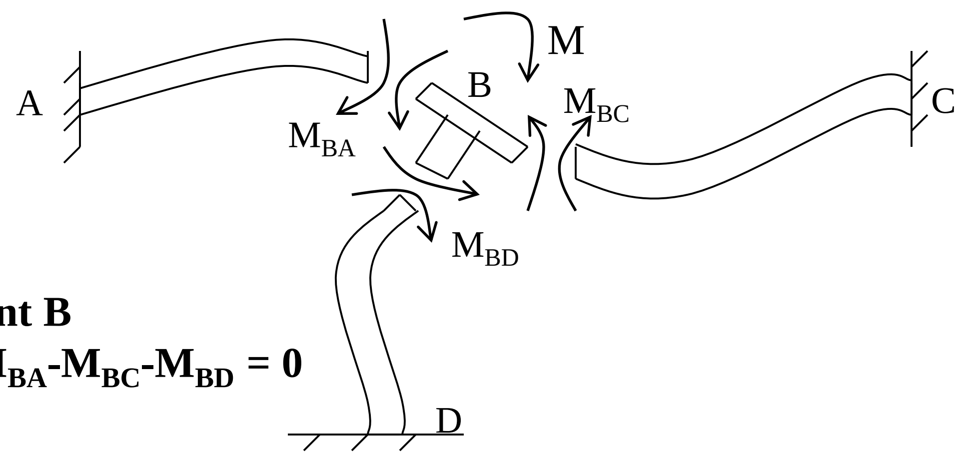
$$M_B + M_A + R_A L = 0$$
$$\therefore M_B = M_A/2 = (1/2)M_A$$

**Carry - over Factor = 1/2**

## 7.3.2 Distribution Factor

Distribution factor is the ratio according to which an externally applied unbalanced moment M at a joint is apportioned to the various members mating at the joint




**At joint B**

$$M - M_{BA} - M_{BC} - M_{BD} = 0$$

**i.e.,** $\mathbf{M = M_{BA} + M_{BC} + M_{BD}}$

$$= \left[ \left( \frac{4E_1 I_1}{L_1} \right) + \left( \frac{4E_2 I_2}{L_2} \right) + \left( \frac{4E_3 I_3}{L_3} \right) \right] \theta_B$$

$$= (K_{BA} + K_{BC} + K_{BD}) \theta_B$$

$$\therefore \quad \theta_B = \frac{M}{(K_{BA} + K_{BC} + K_{BD})} = \frac{M}{\sum K}$$

$$M_{BA} = K_{BA} \theta_B = \left( \frac{K_{BA}}{\sum K} \right) M = (D.F)_{BA} M$$

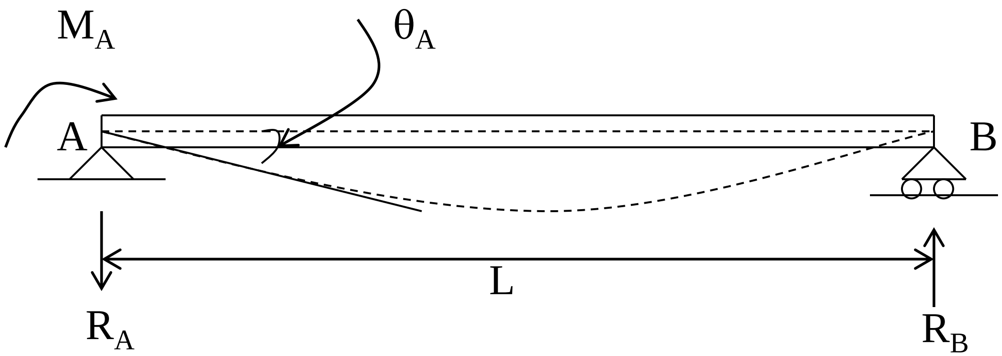
*Similarly*

$$M_{BC} = \left( \frac{K_{BC}}{\sum K} \right) M = (D.F)_{BC} M$$

$$M_{BD} = \left( \frac{K_{BD}}{\sum K} \right) M = (D.F)_{BD} M$$

## 7.3.3 Modified Stiffness Factor

The stiffness factor changes when the far end of the beam is simply-supported.




As per earlier equations for deformation, given in Mechanics of Solids text-books.

$$\theta_A = \frac{M_A L}{3EI}$$

$$K_{AB} = \frac{M_A}{\theta_A} = \frac{3EI}{L} = \left(\frac{3}{4}\right)\left(\frac{4EI}{L}\right)$$

$$= \frac{3}{4}(K_{AB})_{fixed}$$

# 7.4 SOLUTION OF PROBLEMS -

## 7.4.1 Solve the previously given problem by the moment distribution method

### 7.4.1.1: Fixed end moments

$$M_{AB} = -M_{BA} = -\frac{wl^2}{12} = -\frac{(15)(8)^2}{12} = -80 \ kN.m$$

$$M_{BC} = -M_{CB} = -\frac{wl}{8} = -\frac{(150)(6)}{8} = -112.5 \ kN.m$$

$$M_{CD} = -M_{DC} = -\frac{wl^2}{12} = -\frac{(10)(8)^2}{12} = -53.333 \ kN.m$$

### 7.4.1.2 Stiffness Factors (Unmodified Stiffness)

$$\mathbf{K}_{AB} = \mathbf{K}_{BA} = \frac{4\mathbf{EI}}{\mathbf{L}} = \frac{(4)(\mathbf{EI})}{8} = 0.5\mathbf{EI}$$

$$\mathbf{K}_{BC} = \mathbf{K}_{CB} = \frac{4\mathbf{EI}}{\mathbf{L}} = \frac{(4)(\mathbf{EI})}{6} = 0.667\ \mathbf{EI}$$

$$\mathbf{K}_{CD} = \left[\frac{4\mathbf{EI}}{8}\right] = \frac{4}{8}\mathbf{EI} = 0.5\mathbf{EI}$$

$$\mathbf{K}_{DC} = \frac{4\mathbf{EI}}{8} = 0.5\mathbf{EI}$$

## 7.4.1.3 Distribution Factors

$$DF_{AB} = \frac{K_{BA}}{K_{BA} + K_{wall}} = \frac{0.5EI}{0.5 + \infty\ (\text{wall stiffness})} = 0.0$$

$$DF_{BA} = \frac{K_{BA}}{K_{BA} + K_{BC}} = \frac{0.5EI}{0.5EI + 0.667\ EI} = 0.4284$$

$$DF_{BC} = \frac{K_{BC}}{K_{BA} + K_{BC}} = \frac{0.667\ EI}{0.5EI + 0.667\ EI} = 0.5716$$

$$DF_{CB} = \frac{K_{CB}}{K_{CB} + K_{CD}} = \frac{0.667\ EI}{0.667\ EI + 0.500\ EI} = 0.5716$$

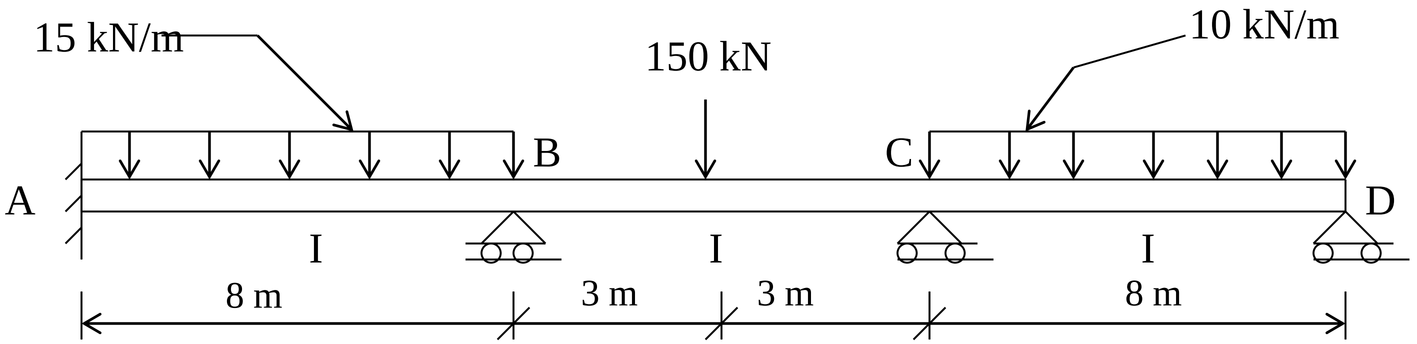
$$DF_{CD} = \frac{K_{CD}}{K_{CB} + K_{CD}} = \frac{0.500\ EI}{0.667\ EI + 0.500\ EI} = 0.4284$$

$$DF_{DC} = \frac{K_{DC}}{K_{DC}} = 1.00$$

## 7.4.1.4 Moment Distribution Table

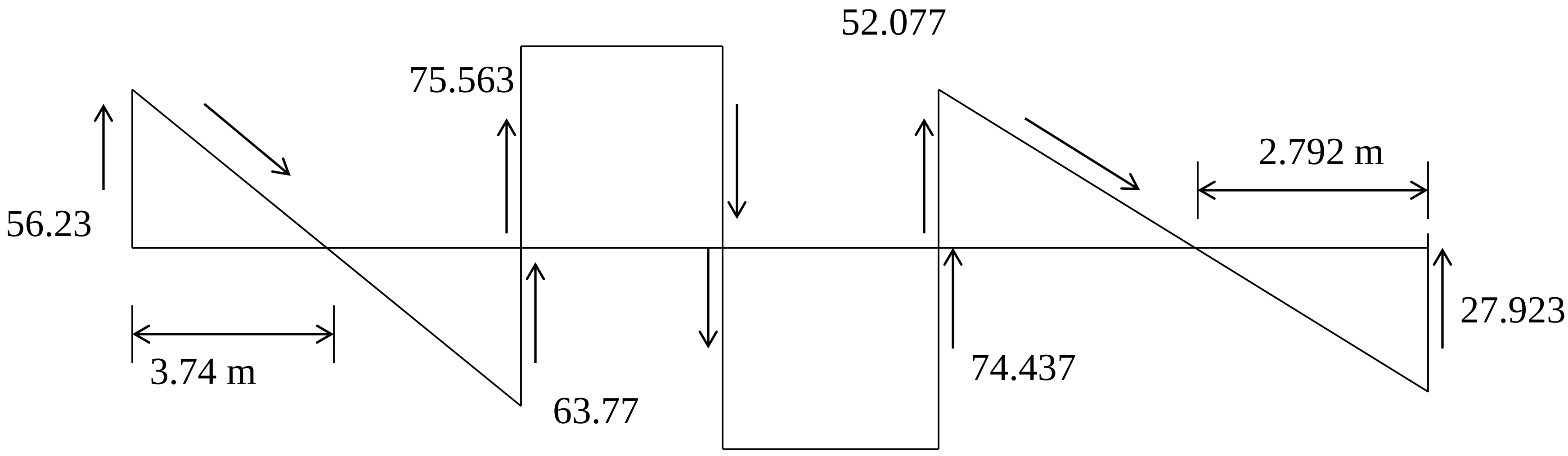
| Joint | | A | B | | C | | D |
|---|---|---|---|---|---|---|---|
| Member | | AB | BA | BC | CB | CD | DC |
| Distribution Factors | | 0 | 0.4284 | 0.5716 | 0.5716 | 0.4284 | 1 |
| Cycle 1 | Computed end moments | -80 | 80 | -112.5 | 112.5 | -53.33 | 53.33 |
| | Distribution | | 13.923 | 18.577 | -33.82 | -25.35 | -53.33 |
| Cycle 2 | Carry-over moments | 6.962 | | -16.91 | 9.289 | -26.67 | -12.35 |
| | Distribution | | 7.244 | 9.662 | 9.935 | 7.446 | 12.35 |
| Cycle 3 | Carry-over moments | 3.622 | | 4.968 | 4.831 | 6.175 | 3.723 |
| | Distribution | | -2.128 | -2.84 | -6.129 | -4.715 | -3.723 |
| Cycle 4 | Carry-over moments | -1.064 | | -3.146 | -1.42 | -1.862 | -2.358 |
| | Distribution | | 1.348 | 1.798 | 1.876 | 1.406 | 2.358 |
| Cycle 5 | Carry-over moments | 0.674 | | 0.938 | 0.9 | 1.179 | 0.703 |
| | Distribution | | -0.402 | -0.536 | -1.187 | -0.891 | -0.703 |
| | Summed up moments | -69.81 | 99.985 | -99.99 | 96.613 | -96.61 | 0 |

# 7.4.1.5 Computation of Shear Forces

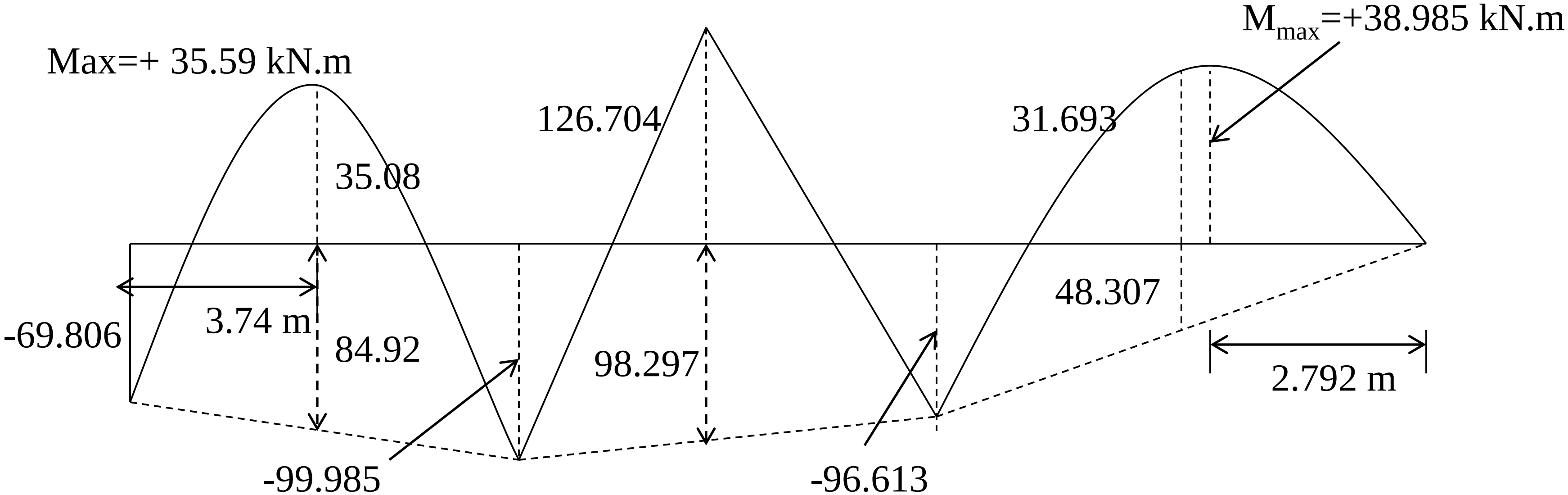



| | | | | | | |
|---|---|---|---|---|---|---|
| Simply-supported reaction | 60 | 60 | 75 | 75 | 40 | 40 |
| End reaction due to left hand FEM | 8.726 | -8.726 | 16.665 | -16.67 | 12.079 | -12.08 |
| End reaction due to right hand FEM | -12.5 | 12.498 | -16.1 | 16.102 | 0 | 0 |
| Summed-up moments | 56.228 | 63.772 | 75.563 | 74.437 | 53.077 | 27.923 |

## 7.4.1.5 Shear Force and Bending Moment Diagrams




**S. F. D.**




**B. M. D**

## Simply-supported bending moments at center of span

**$M_{center}$ in AB = $(15)(8)^2/8 = +120$ kN.m**

**$M_{center}$ in BC = $(150)(6)/4 = +225$ kN.m**

**$M_{center}$ in AB = $(10)(8)^2/8 = +80$ kN.m**