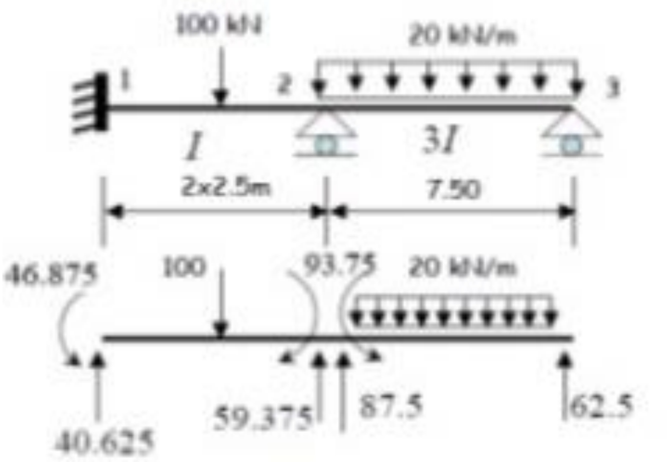
ANALYSIS
OF
INDETERMINATE STRUCTURE
BY
MOMENT DISTRIBUTION METHOD

Example: It is required to determine the support moments for the continuous beam.



$$-M_{12}^F = M_{21}^F = \frac{100*5}{8} = 62.5kNm$$

$$-M_{23}^F = M_{32}^F = \frac{20*7.5^2}{12} = 93.75kNm$$

Slope – Deflection Equations

$$M_{12} = \frac{2EI}{5}\theta_2 - 62.5 =$$

$$M_{21} = \frac{2EI}{5}2\theta_2 + 62.5 =$$

$$M_{23} = \frac{6EI}{7.5}(2\theta_2 + \theta_3) - 93.75 =$$

$$M_{32} = \frac{6EI}{7.5}(\theta_2 + 2\theta_3) + 93.75 =$$

Equilibrium equations of joints

$M_{21} + M_{23} = 0$

$M_{32} = 0$

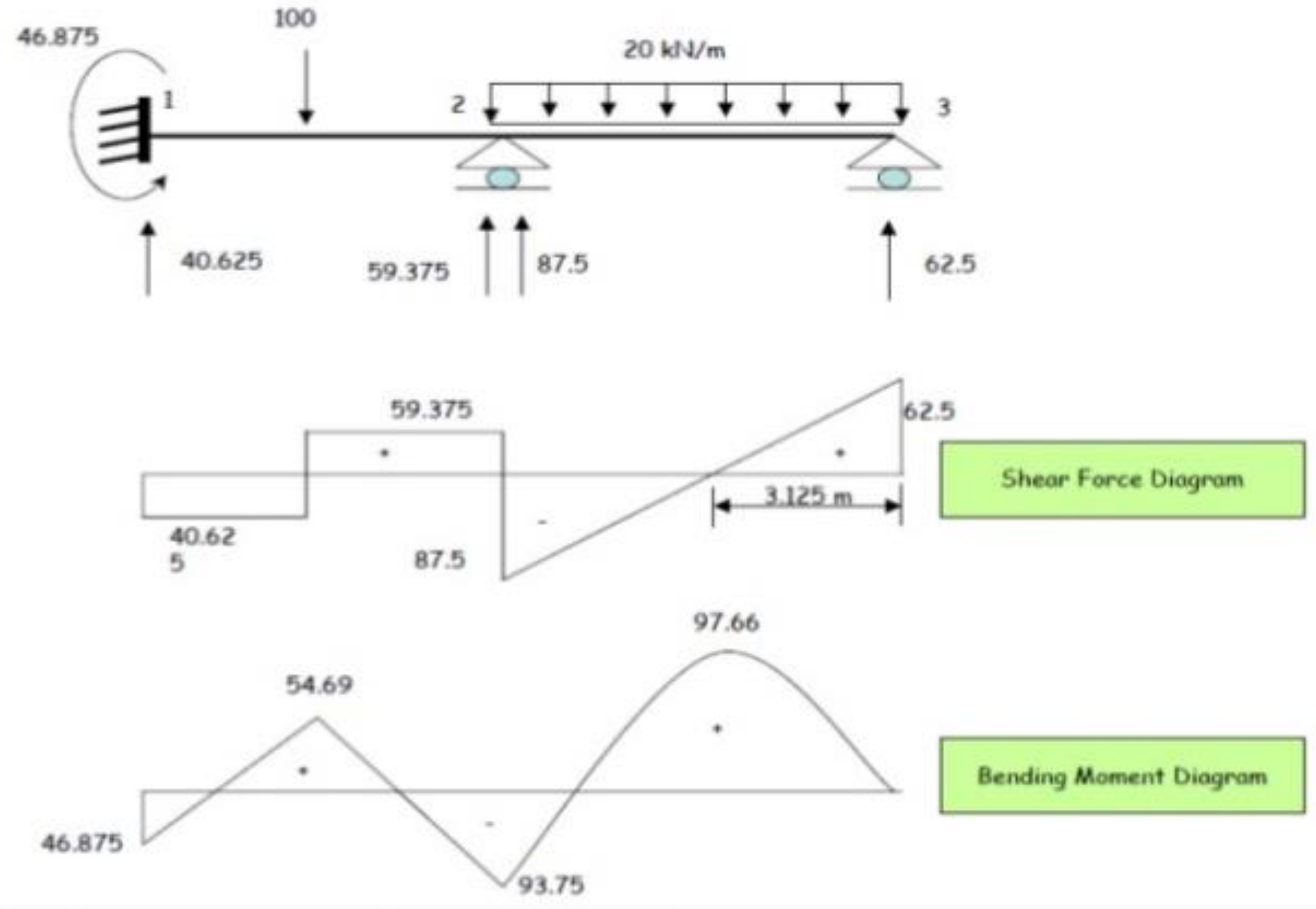
$$2.4EI\theta_2 + 0.8EI\theta_3 = 31.25$$

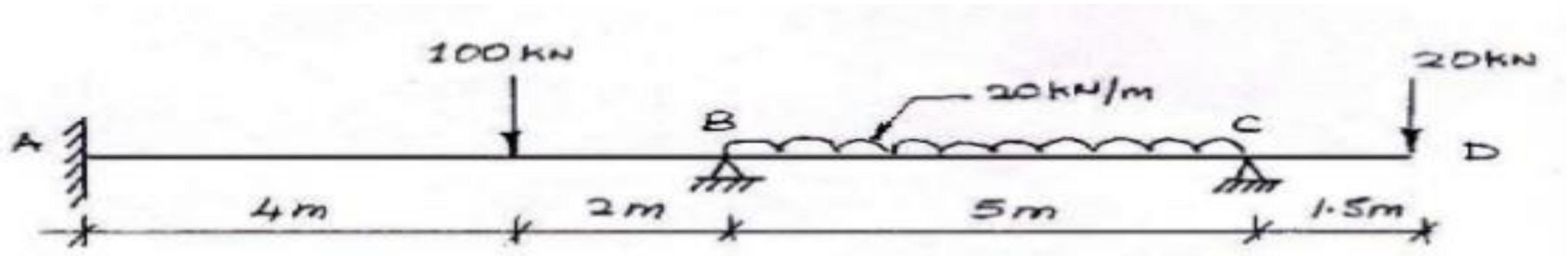
$$0.8EI\theta_2 + 1.6EI\theta_3 = -93.75 \rightarrow \theta_2 = \frac{39.0625}{EI} \rightarrow \theta_3 = \frac{-78.125}{EI}$$

Substitude these results in slope deflection equations

$M_{12} = -46.875kNm, \rightarrow M_{21} = 93.75kNm$

$M_{23} = -93.75kNm, \rightarrow M_{32} = 0kNm$


46.875
100
20 kN/m
1
2
3
40.625
59.375
87.5
62.5
59.375
62.5
Shear Force Diagram
3.125 m
40.62
5
87.5
97.66
54.69
Bending Moment Diagram
46.875
93.75



Solution:

$$\theta_A = 0, \ \theta_B \neq 0, \ \theta_C \neq 0$$

FEMS

$$F_{AB} = -\frac{Wab^2}{L^2} = -\frac{100 \times 4 \times 2^2}{6^2} = -44.44 \ \text{KNM}$$

$$F_{BA} = +\frac{Wa^2b}{L^2} = +\frac{100 \times 4^2 \times 2}{6^2} = +88.88 \ \text{KNM}$$

$$F_{BC} = -\frac{wL^2}{12} = -\frac{20 \times 5^2}{12} = -41.67 \ \text{KNM}$$

$$F_{CB} = +\frac{wL^2}{12} = +\frac{20 \times 5^2}{12} = +41.67 \ \text{KNM}$$

$$F_{CD} = -20 \times 1.5 = -30 \ \text{KNM}$$

Slope deflection equations:

$$M_{AB}=F_{AB}+\frac{2EI}{L}(2\theta_A+\theta_B)=-44.44+\frac{1}{3}EI\theta_B \qquad \text{-------->}(1)$$

$$M_{BA}=F_{BA}+\frac{2EI}{L}(2\theta_B+\theta_A)=+88.89+\frac{2}{3}EI\theta_B \qquad \text{-------->}(2)$$

$$M_{BC}=F_{BC}+\frac{2EI}{L}(2\theta_B+\theta_C)=-41.67+\frac{4}{5}EI\theta_B+\frac{2}{5}EI\theta_C \qquad \text{-------->}(3)$$

$$M_{CB}=F_{CB}+\frac{2EI}{L}(2\theta_C+\theta_B)=+41.67+\frac{4}{5}EI\theta_C+\frac{2}{5}EI\theta_B \qquad \text{-------->}(4)$$

$$M_{CD}=-30\ \text{KNM}$$

In the above equations we have two unknown rotations θ_B and θ_C, accordingly the boundary conditions are:

$$M_{BA} + M_{BC} = 0$$
$$M_{CB} + M_{CD} = 0$$

Now, $$M_{BA} + M_{BC} = 88.89 + \frac{2}{3}EI\theta_B - 41.67 + \frac{4}{5}EI\theta_B + \frac{2}{5}EI\theta_C$$
$$= 47.22 + \frac{22}{15}EI\theta_B + \frac{2}{5}EI\theta_C = 0 \quad \text{--------->(5)}$$

And, $$M_{CB} + M_{CD} = +41.67 + \frac{4}{5}EI\theta_C + \frac{2}{5}EI\theta_B - 30$$
$$= 11.67 + \frac{2}{5}EI\theta_B + \frac{4}{5}EI\theta_C \quad \text{------->(6)}$$

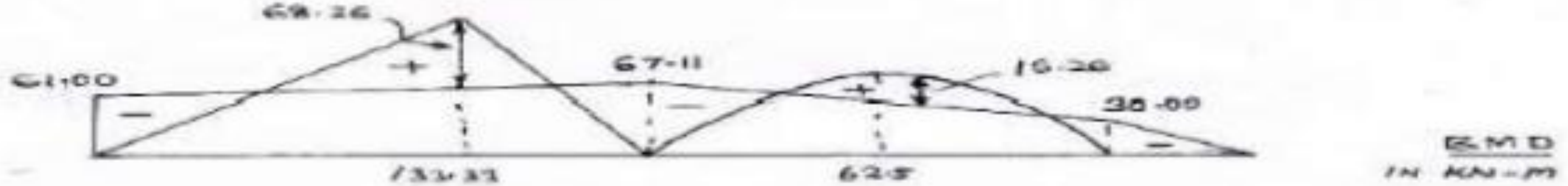
Solving (5) and (6) we get

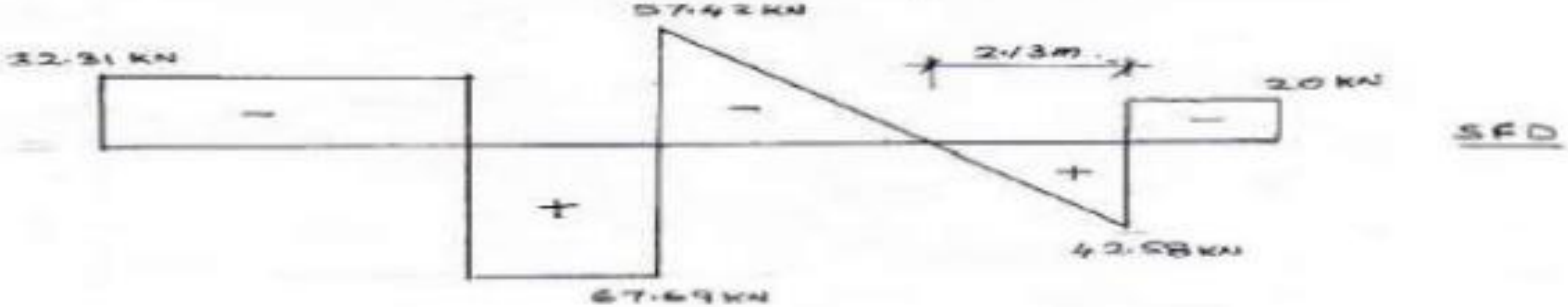
$EI\theta_B = -32.67$ Rotation @ B anticlockwise
$EI\theta_C = +1.75$ Rotation @ B clockwise

Substituting value of $EI\theta_B$ and $EI\theta_C$ in slope deflection equations we have

$$M_{AB} = -44.44 + \frac{1}{2}(-32.67) = -61.00 \text{ KNM}$$
$$M_{BA} = +88.89 + \frac{2}{3}(-32.67) = +67.11 \text{ KNM}$$
$$M_{BC} = -41.67 + \frac{4}{5}(-32.67) = +\frac{2}{5}(1.75) = -67.11 \text{ KNM}$$
$$M_{CB} = +41.67 + \frac{4}{5}(1.75) + \frac{2}{5}(-32.67) = +30.00 \text{ KNM}$$
$$M_{CD} = -30 \text{ KNM}$$

Reactions: Consider free body diagram of beam AB, BC and CD as shown





Span AB

$$R_B \times 6 = 100 \times 4 + 67.11 - 61$$
$$R_B = 67.69 \quad KN$$
$$R_A = 100 - R_B = 32.31 \quad KN$$

Span BC

$$R_C \times 5 = 20 \times \frac{5}{2} \times 5 + 30 - 67.11$$
$$R_C = 42.58 \quad KN$$
$$R_B = 20 \times 5 - R_B = 57.42 \quad KN$$

Maximum Bending Moments:

Span AB: Occurs under point load

$$Max = 133.33 - 61 - \left(\frac{67.11 - 61}{6} \times 4 = 68.26 \quad KNM \right)$$

Span BC: where SF=0, consider SF equation with C as reference

$$S_x = 42.58 - 20x = 0$$
$$x = \frac{42.58}{20} = 2.13 \quad m$$
$$\therefore M_{max} = 42.58 \times 2.13 - 20 \times \frac{2.13^2}{2} - 30 = 15.26 \quad KNM$$

THANK YOU