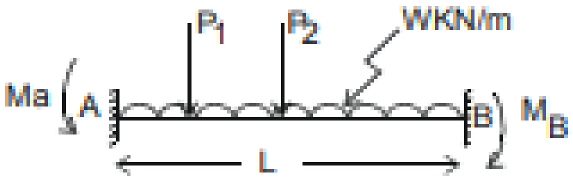
# COLUMN ANALOGY METHOD

The column analogy method was also proposed by Prof. Hardy Cross and is a powerful technique to analyze the beams with fixed supports, fixed ended gable frames, closed frames & fixed arches etc., These members may be of uniform or variable moment of inertia throughout their lengths but the method is ideally suited to the calculation of the stiffness factor and the carryover factor for the members having variable moment of inertia. The method is strictly applicable to a maximum of 3rd degree of indeterminacy. This method is essentially an indirect application of the consistent deformation method.

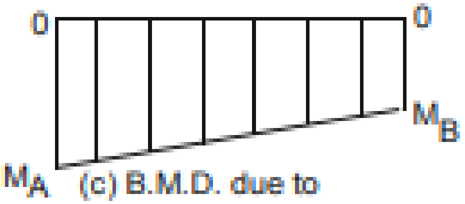
The method is based on a mathematical similarity (i.e. analogy) between the stresses developed on a column section subjected to eccentric load and the moments imposed on a member due to fixity of its supports. *(We have already used an analogy in the form of method of moment and shear in which it was assumed that parallel chord trusses behave as a deep beam). In the analysis of actual engineering structures of modern times, so many analogies are used like slab analogy, and shell analogy etc. In all these methods, calculations are not made directly on the actual structure but, in fact it is always assumed that the actual structure has been replaced by its mathematical model and the calculations are made on the model. The final results are related to the actual structure through same logical engineering interpretation.

In the method of column analogy, the actual structure is considered under the action of applied loads and the redundants acting simultaneously on a BDS. The load on the top of the analogous column is usually the B.M.D. due to applied loads on simple spans and therefore the reaction to this applied load is the B.M.D. due to redundants on simple spans considers the following fixed ended loaded beam.

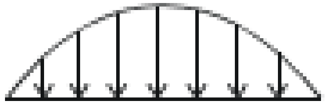



EI=Constt.
(a) Given beam under loads

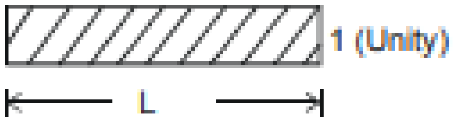

(b) B.M.D. due to applied loads, on simple span plotted on the compressin side.



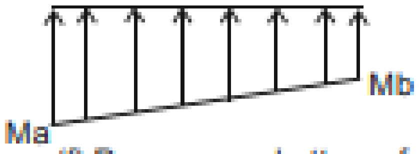

(c) B.M.D. due to redundants, plotted on the compression side on simple span



(d) Loading on top of analogous column, Ms diagram, same as(b).




(e) X-section of analogous column.




(f) Pressure on bottom of analogous column, Mi diagram.

The resultant of B.M.D's due to applied loads does not fall on the mid point of analogous column section which is eccentrically loaded.

Msdiagram = BDS moment diagram due to applied loads.

Mi diagram = Indeterminate moment diagram due to redundants.

If we plot (+ve) B.M.D. above the zero line and (–ve) B.M.D below the zero line (both on compression sides due to two sets of loads) then we can say that these diagrams have been plotted on the compression side.

(The conditions from which $M_A$ & $M_B$ can be determined, when the method of consistent deformation is used, are as follows). From the Geometry requirements, we know that

(1) The change of slope between points A & B = 0; or sum of area of moment diagrams between A & B = 0 (note that EI = Constt:), or area of moment diagrams of fig.b = area of moment diagram of fig..c.

(2) The deviation of point B from tangent at A = 0; or sum of moment of moment diagrams between A & B about B = 0, or Moment of moment diagram of fig.(b) about B = moment of moment diagram of fig.(c) about B. Above two requirements can be stated as follows.

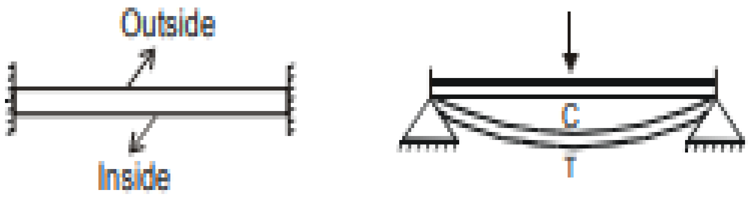
   (1) Total load on the top is equal to the total pressure at the bottom and;

   (2) Moment of load about B is equal to the moment of pressure about B), indicates that the analogous column is on equilibrium under the action of applied loads and the redundants.

# SIGN CONVENTIONS:–

It is necessary to establish a sign convention regarding the nature of the applied load (Ms – diagram) and the pressures acting at the base of the analogous column (Mi–diagram.)

1. Load ( P) on top of the analogous column is downward if Ms/EI diagram is (+ve) which means that it causes compression on the outside or (sagging) in BDS vice-versa. If EI is constant, it can be taken equal to units.




2. Upward pressure on bottom of the analogous column ( Mi – diagram) is considered as (+ve).

3. Moment (M) at any point of the given indeterminate structure ( maximum to 3rd degree) is given by the formula.

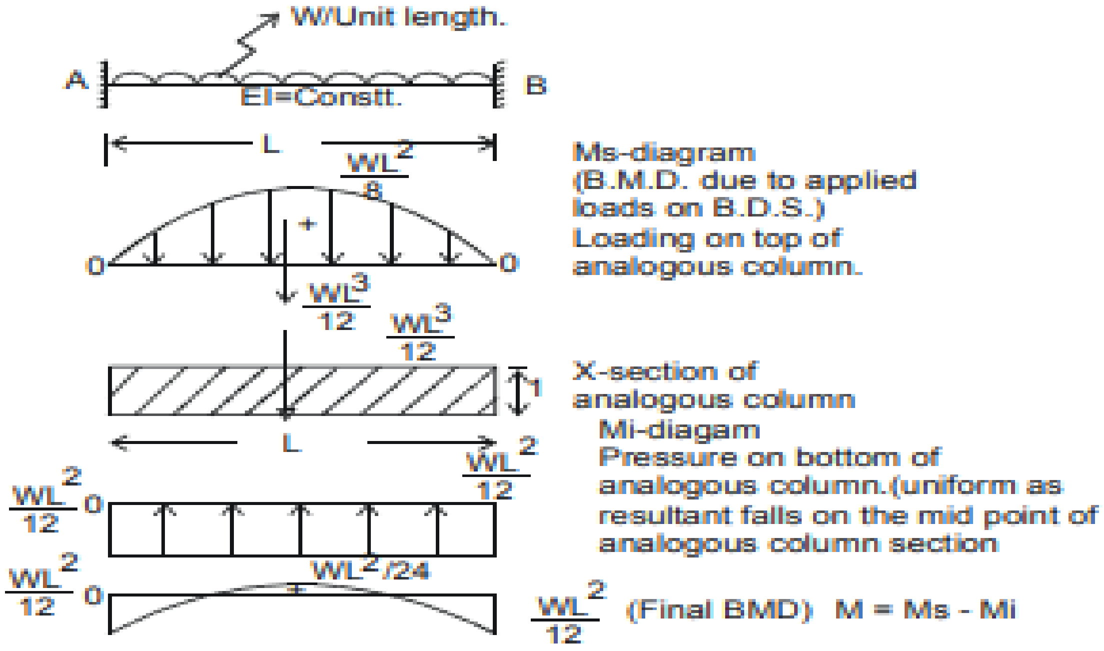
$$M = Ms - Mi,$$

which is (+ve) if it causes compression on the outside of members.

**EXAMPLE NO. 1:–** Determine the fixed–ended moments for the beam shown below by the method of column analogy.

**SOLUTION:**

Choosing BDS as a simple beam. Draw Ms diagram. Please it on analogous column.




Pressure at the base of the column $= \frac{P}{A}$

$A = L \times 1$ (area of analogous column section).

$$= \frac{WL^3}{12(L x 1)}$$

$$Mi = \frac{WL^2}{12}$$ . In this case, it will be uniform as resultant of Ms diagram falls on centroid of analogous column)

$(MS)a = 0$ , (Ms at point A to be picked up for M-s diagram)

$$Ma = (Ms - Mi)_a \quad , \quad \text{(net moment at point A)}$$

$$= 0 - \frac{WL^2}{12}$$

$$Ma = -\frac{WL^2}{12}$$

$$Mb = (Ms-Mi)b = \left(0 - \frac{WL^2}{12}\right) = \frac{-WL^2}{12}$$

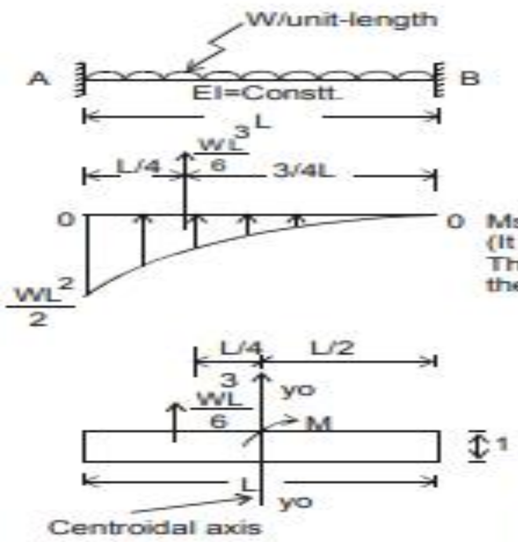
$$Mc = (Ms - Mi)c = \frac{WL^2}{8} - \frac{WL^2}{12}$$

$$Mc = \frac{3\,WL^2 - 2\,WL^2}{24} = \frac{WL^2}{24}$$

. Plot these values to get M = Ms – Mi diagram.

The beam has been analyzed.

**EXAMPLE NO. 2:- SOLVING THE PREVIOUS EXAMPLE, IF B.D.S. IS A CANTILEVER SUPPORTED AT 'A'.**




Ms-diagram
(It creates hagging so load acts upwards)
The resultant of Ms diagram does not fall on the centroid of analogous column.

X-section of analogous column. Carrying eccentric load of $WL^3/6$

Eccentric load $wL^3/6$ acts on centre of analogous column x-section with an associated moment as well
(Eccentric load = Concentric load plus accomprying moment)

Area of Ms diagram $\quad A = \frac{bh}{(n+1)} = \frac{L \times WL^2}{2(2+1)} = \frac{WL^3}{6}$

$$X' = \frac{b}{(n+2)} = \frac{L}{(2+2)} = \frac{L}{4} \qquad \text{(from nearest and)}$$

Alternatively centroid can be located by using the following formula)

$$\bar{X} = \frac{\int MXdX}{\int MdX}$$

$$\int MdX = \int_0^L \left(-\frac{WX^2}{2}\right) dX = -\frac{W}{2} \left|\frac{X^3}{3}\right|_0^L = -\frac{WL^3}{6} \text{ ( Same as above)}$$

$$\int MXdX = \int_0^L \left(-\frac{WX^2}{2}\right) XdX = \int_0^L -\frac{WX^3}{2} dx$$

$$= -\frac{W}{2} \left|\frac{X^4}{4}\right|_0^L = -\frac{WL^4}{8}$$

$$\bar{X} = \frac{\int MXdX}{\int MdX}$$

$$\bar{X} = \frac{-WL^4}{8} \times \frac{6}{(-WL^3)} = \frac{3}{4}L.$$ (from the origin of moment expression or from farthest end)

**NOTE :** Moment expression is always independent of the variation of inertia.

## Properties of Analogous Column X–section :–

1. Area of analogous column section, $A = L \times 1 = L$
2. Moment of inertia, $I\,y_o y_o = \frac{L^3}{12}$
3. Location of centroidal column axis, $C = \frac{L}{2}$

$A\,e' = M = \left(\frac{WL^3}{6}\right)\left(\frac{L}{4}\right) = \frac{WL^4}{24}$ , ( $\frac{L}{4}$ is distance between axis yo– yo and the centroid of Ms diagram where the load equal to area of Ms diagram acts.)

$$(Mi)a = \frac{P}{A} \pm \frac{Mc}{I}$$ (P is the area of Ms diagram and is acting upwards so negative $C = \frac{L}{2}$ and $I = \frac{L^3}{12}$)

$$= \frac{-WL^3}{6\,.\,L} - \frac{WL^4\,.\,L\,.\,12}{24\,.\,2\,.\,L^3}$$ (Load P on analogous column is negative)

$$= \frac{-WL^2}{6} - \frac{WL^2}{4}$$ ( Reaction due to MC/I would be having the same direction at A as that due to P while at B these two would be opposite)

$$= \frac{-2WL^2 - 3\,WL^2}{12}$$

$$= \frac{-5}{12}WL^2$$

$$(Ms)a = \frac{-WL^2}{2}$$

$$Ma = (Ms - Mi)a$$

$$= \frac{-WL^2}{2} + \frac{5}{12} WL^2$$

$$= \frac{-6\ WL^2 + 5\ WL^2}{12}$$

$$Ma = \frac{-WL^2}{12}$$

$$Mb = (Ms - Mi)b$$

$$(Mi)b = \frac{P}{A} \pm \frac{Mc}{I}$$

$$= \frac{-WL^3}{6 \times L} + \frac{WL^4 \times L \times 12}{24 \times 2 \times L^3}$$

$$= \frac{-WL^2}{6} + \frac{WL^2}{4}$$

$$= \frac{-2WL^2 + 3\ WL^2}{12}$$

$$= \frac{WL^2}{12}$$

$$(Ms)b = 0$$

$$Mb = (Ms - Mi)b = 0 - \frac{WL^2}{12} = -\frac{WL^2}{12}$$

Same results have been obtained but effort / time involved is more for this BDS).