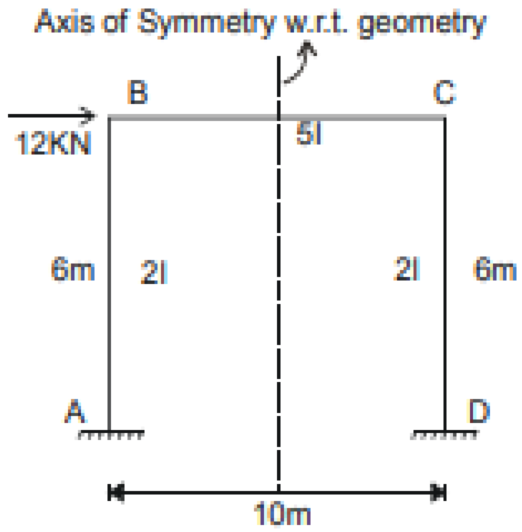
APPLICATION TO FRAMES WITH ONE AXIS OF SYMMETRY:–

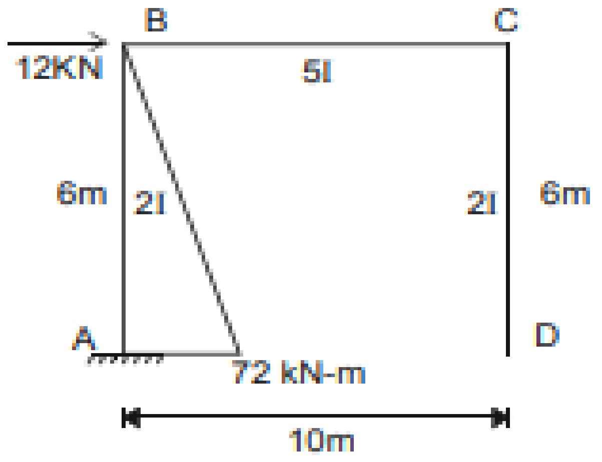
EXAMPLE NO. 9:- Analyze the quadrangular frame shown below by the method of column analogy. Check the solution by using a different B.D.S.

SOLUTION:–

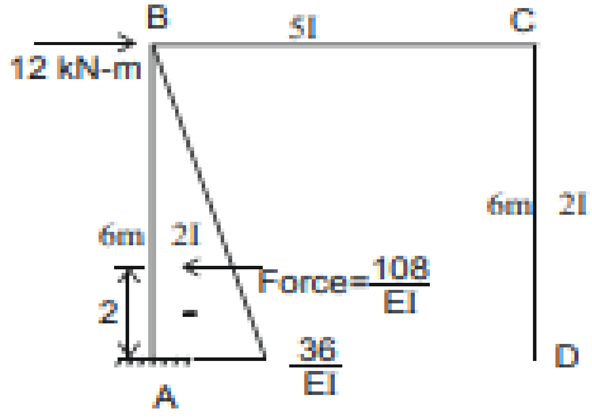


The term "axis of symmetry" implies that the shown frame is geometrically symmetrical (M.O.I. and support conditions etc., are symmetrical) w.r.t. one axis as shown in the diagram. The term does not include the loading symmetry (the loading can be and is unsymmetrical).

Choosing the B.D.S. as a cantilever supported. at A.

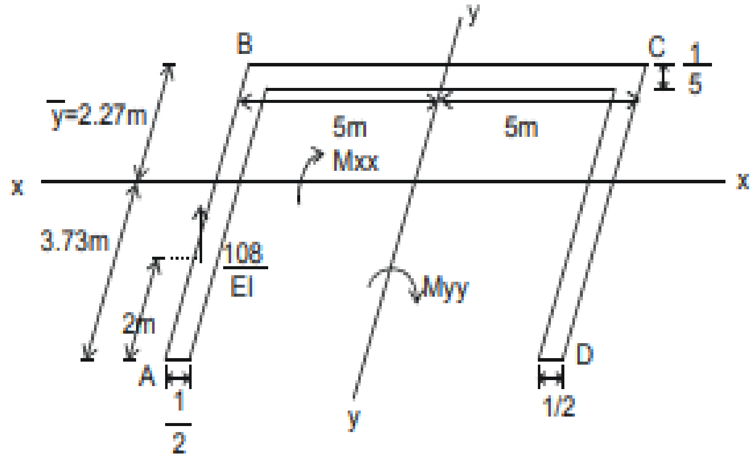


Ms-diagram



$\frac{Ms}{EI}$ - Diagram

According to our sign convention for column analogy, the loading arising out of negative $\frac{Ms}{EI}$ giving tension on outside will act upwards on the analogous column section. Sketch analogous column section and place load.



(1) Properties of Analogous Column Section:–

$$A = \left(\frac{1}{2} \times 6\right) \times 2 + \frac{1}{5} \times 10 = \frac{8}{EI}$$

$$\bar{y} = \left[\frac{\left(\frac{1}{5} \times 10\right) \times \frac{1}{10} + 2\left[\frac{1}{2} \times 6 \times 3\right]\frac{1}{EI}}{\frac{8}{EI}}\right] = 2.27 \text{ m about line BC. (see diagram)}$$

$$Ixx = 2\left[\frac{0.5 \times 6^3}{12} + \left(\frac{1}{2} \times 6\right) x\,(0.73)^2\right] + \frac{10 \times (1/5)^3}{12} + (0.2 \times 10) \times (2.27)^2$$

$$= \frac{31.51}{EI} m^4$$

$$Iyy = \frac{0.2 \times 10^3}{12} + 2\left[\frac{6 \times 0.5^3}{12} + (6 \times 0.5) \times (5)^2\right]$$

$$= \frac{167}{EI} m^4$$

$$Mxx = 108 \times 1.73 = \frac{187}{EI} \text{ clockwise.}$$

$$Myy = 108 \times 5 = \frac{540}{EI} \text{ clockwise.}$$

Applying the formulae in a tabular form for all points. Imagine the direction of reactions at exterior frame points due to loads and moments.

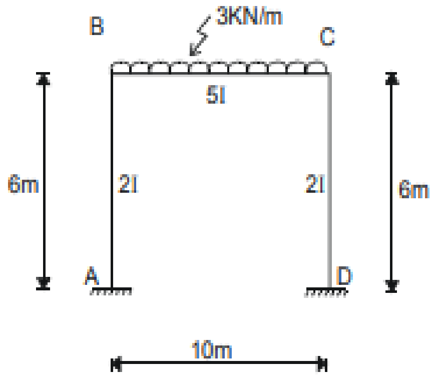
$Ma = (Ms - Mi)a$

$$(Mi)a = \frac{P}{A} \pm \frac{M_x y}{Ix} \pm \frac{M_y X}{Iy}$$

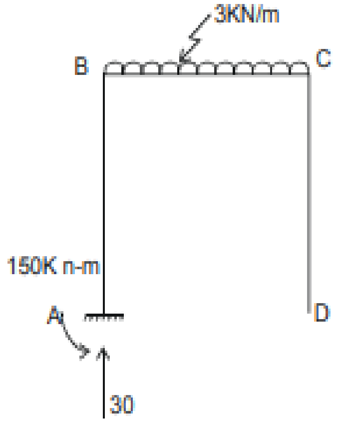
POINT	Ms	P/A	$\frac{Mx\ y}{Ix}$	$\frac{My\ X}{Iy}$	Mi	M = Ms–Mi
A	– 72	– 13.5	– 22.14	– 16.17	– 51.81	– 20.19
B	0	– 13.5	+ 13.47	– 16.17	– 16.20	+ 16.20
C	0	– 13.5	+ 13.47	+ 16.17	+ 16.14	– 16.14
D	0	– 13.5	– 22.14	+ 16.17	– 19.47	+ 19.47

Note: Imagine the direction of reaction due to P, Mx and My at all points A, B, C and P. Use appropriate signs. Repeat the analysis by choosing a different BDS yourself.

EXAMPLE NO. 10:– Analyze the quadrangular frame shown by the method of column analogy.



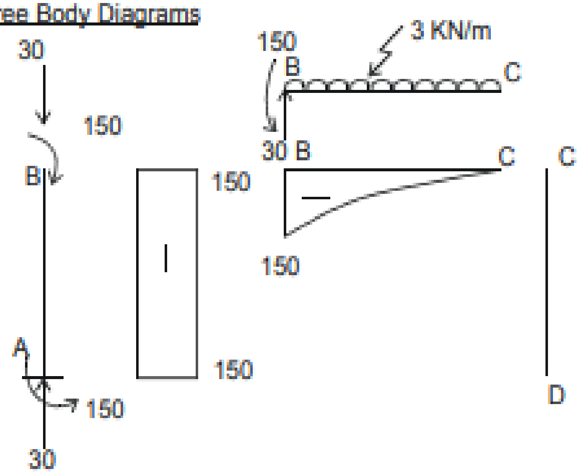
Choosing B.D.S. as a cantilever supported at A.

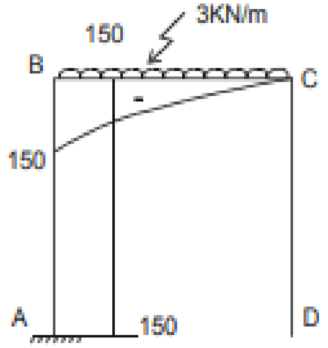


BDS under loads

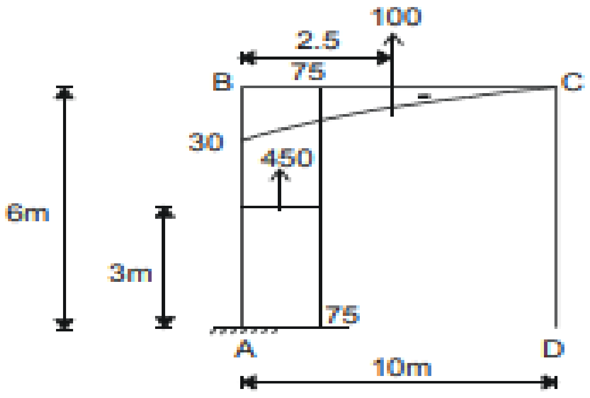
Draw Ms–diagram by parts and then superimpose for convenience and clarity.

Free Body Diagrams





Ms-Diagram



$\frac{Ms}{EI}$ - Diagram

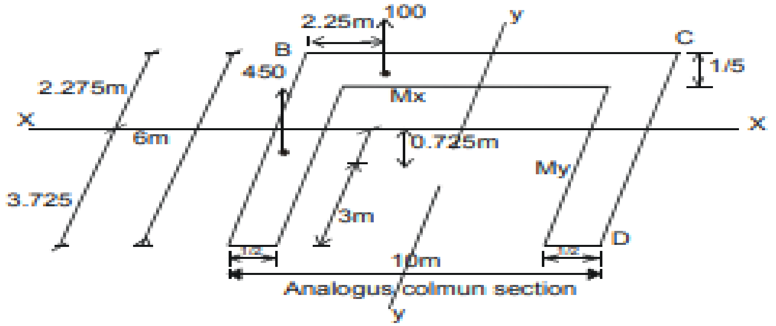
For Portion BC

$$\text{Area} = \frac{bb}{n+1} = \frac{10 \times 30}{2+1} = \frac{300}{3} = 100$$

$$X' = \frac{b}{n+2} = \frac{10}{2+2} = \frac{10}{4} = 2.5 \text{ from B.}$$

Note: As BMD on portions BC and AB are negative the loads equal to their areas will act upwards.

Now sketch analogous column section carrying loads arising from $\frac{M}{EI}$ contributions.



Analogus colmun section

Properties of analogous column section:—

$$A = 2\left[\frac{1}{2} \times 6\right] + \frac{1}{5} \times 10 = \frac{8}{EI} \text{ (as before)}$$

Properties of analogous column section:—

$$A = 2\left[\frac{1}{2} \times 6\right] + \frac{1}{5} \times 10 = \frac{8}{EI} \text{ (as before)}$$

$$\bar{y} = \frac{\left(\frac{1}{5} \times 10\right) \times \frac{1}{10} + 2\left[\left(6 \times \frac{1}{2}\right) \times 3\right]}{8} = 2.275 \text{ about line BC (as before)}$$

$$Ix = 2\left[\frac{1}{2} \times 6^3 + \left(\frac{1}{2} \times 6\right) \times (0.725)^2\right] + \left[10 \times \left(\frac{1}{5}\right)^3 + \left(10 \times \frac{1}{5}\right) \times (2.275)^2\right]$$

$$= \frac{31.51}{EI} m^4 \quad \text{(as before)}$$

$$Iy = 2\left[\frac{6 \times 0.5^3}{12} + (6 \times 0.5) \times 5^2\right] + \frac{0.2 \times 10^3}{12}$$

$$= \frac{166.79}{EI} m^4 \quad \text{(as before)}$$

Mx = 450 × 0.725 – 100 × 2.275 = 95.75 KN–m Clockwise

My = 450 × 5 + 100 × 2.75 = 2525 KN–m clockwise.

P = 100 + 450 = 550 KN

Now this eccentric load P and Mx and My are placed on column centroid.

Applying the formulae in a tabular form.

Ma = (Ms– Mi)a

and $$(Mi)a = \frac{P}{A} \pm \frac{M_x\, y}{Ix} \pm \frac{M_y\, x}{Iy}$$

POINT	Ms	P/A	$\frac{Mx \cdot y}{Ix}$	$\frac{My \cdot x}{Iy}$	Mi	M = Ms–Mi
A	– 150	– 68.75	– 11.32	– 75.69	– 155.76	5.76
B	– 150	–68.75	+ 6.91	– 75.69	– 137.53	–12.47
C	0	–68.75	+ 6.91	+ 75.69	13.85	–13.85
D	0	–68.75	–11.32	+ 75.69	–4.38	4.38