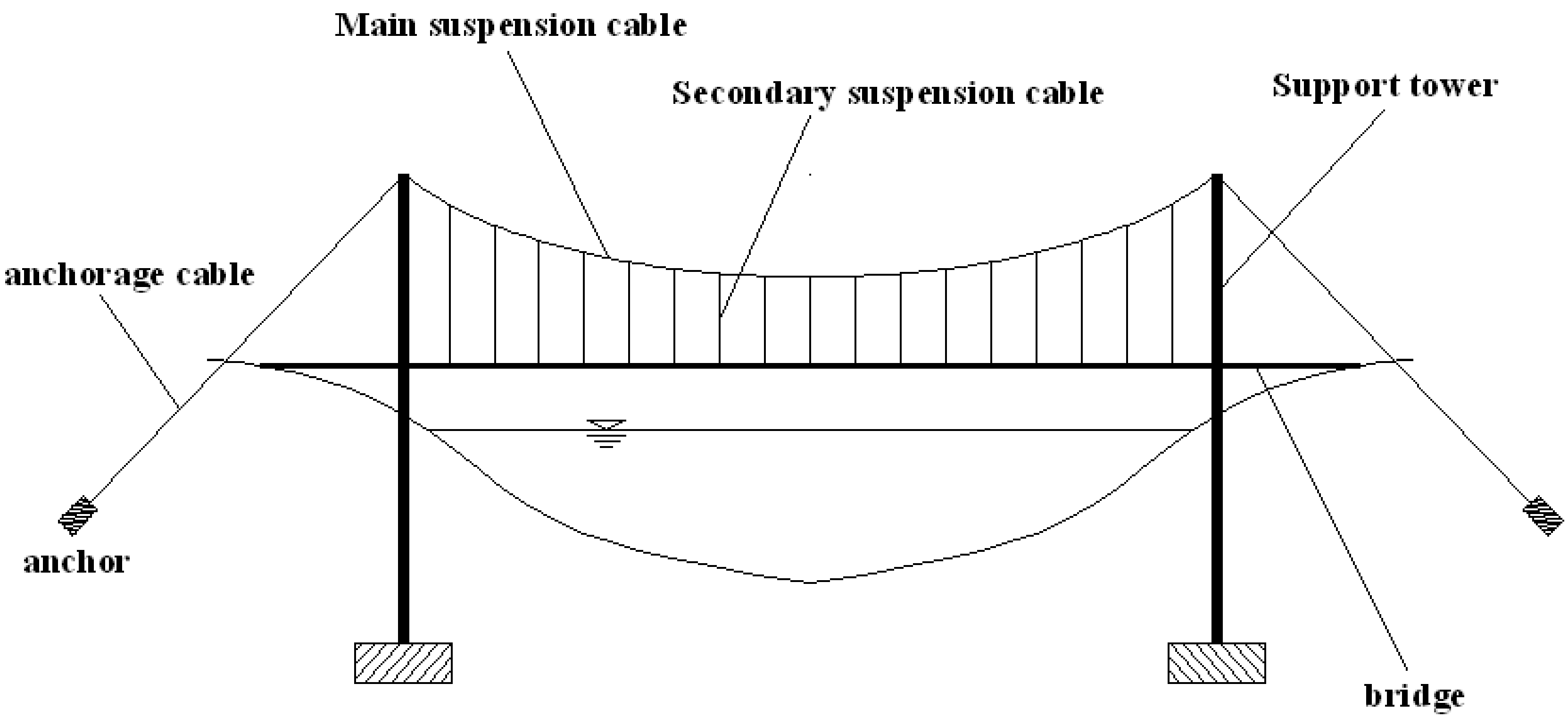
Cable Structure

Cables are often used in engineering structures for support and to transmit loads from one member to another. When used to support suspension bridges, cables form the main load-carrying element in the structure.



Note: Cable can only carrying tensile force.

Parabolic equation of Cable

$$y = kx(L - x)$$

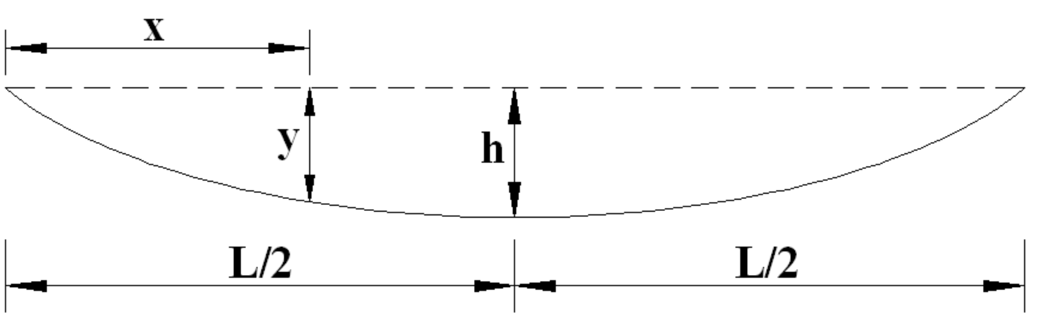
$$x = \frac{L}{2} \qquad y = h$$

$$h = k\frac{L}{2}(L - \frac{L}{2})$$

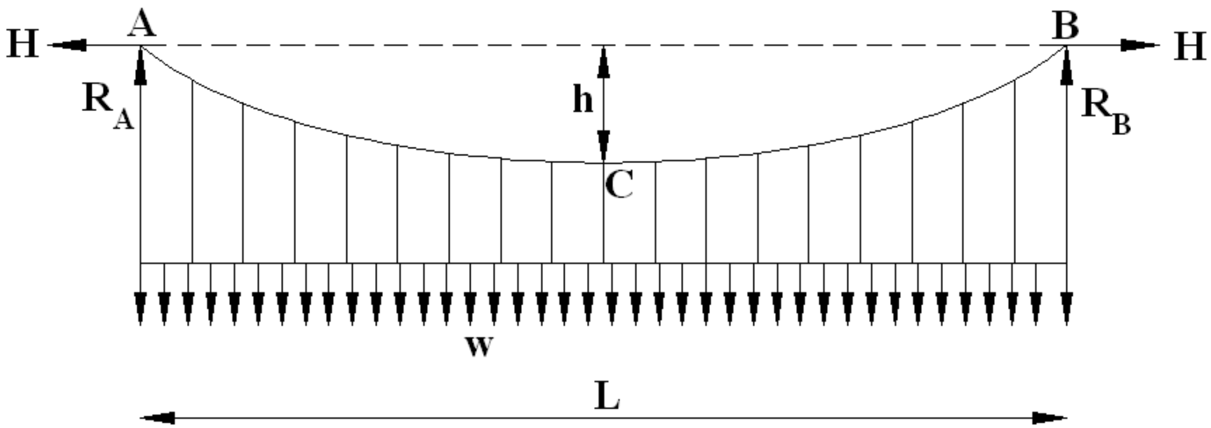
$$h = k\frac{L^2}{4}$$

$$k = \frac{4h}{L^2}$$

$$y = \frac{4hx(L - x)}{L^2}$$



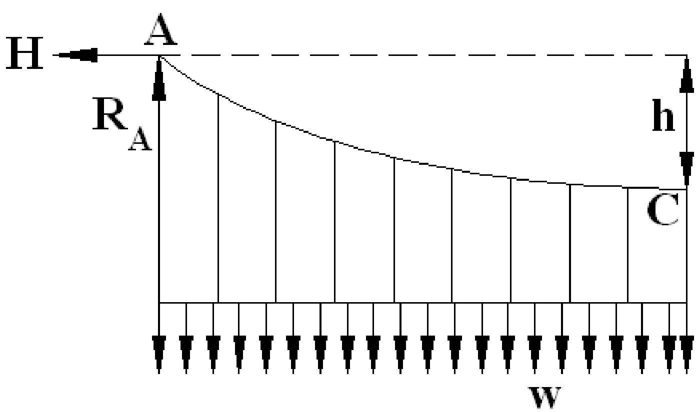
Cable supported at same level



$$\sum F_y = 0 :$$

$$R_A = R_B = \frac{wL}{2}$$

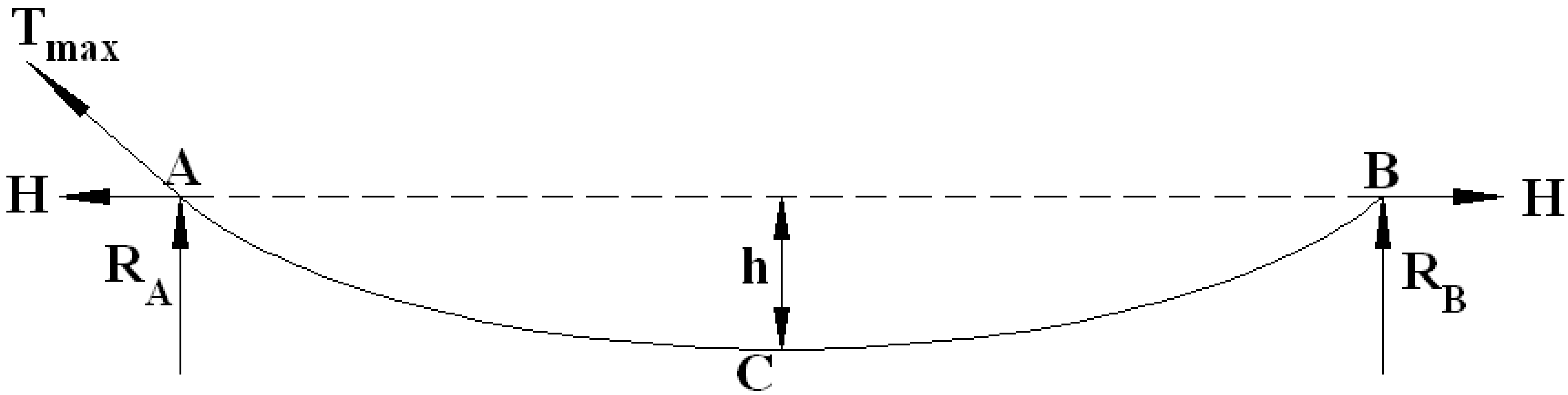
Consider free body diagram at left hand side of C:



$$\sum M_C = 0 :$$

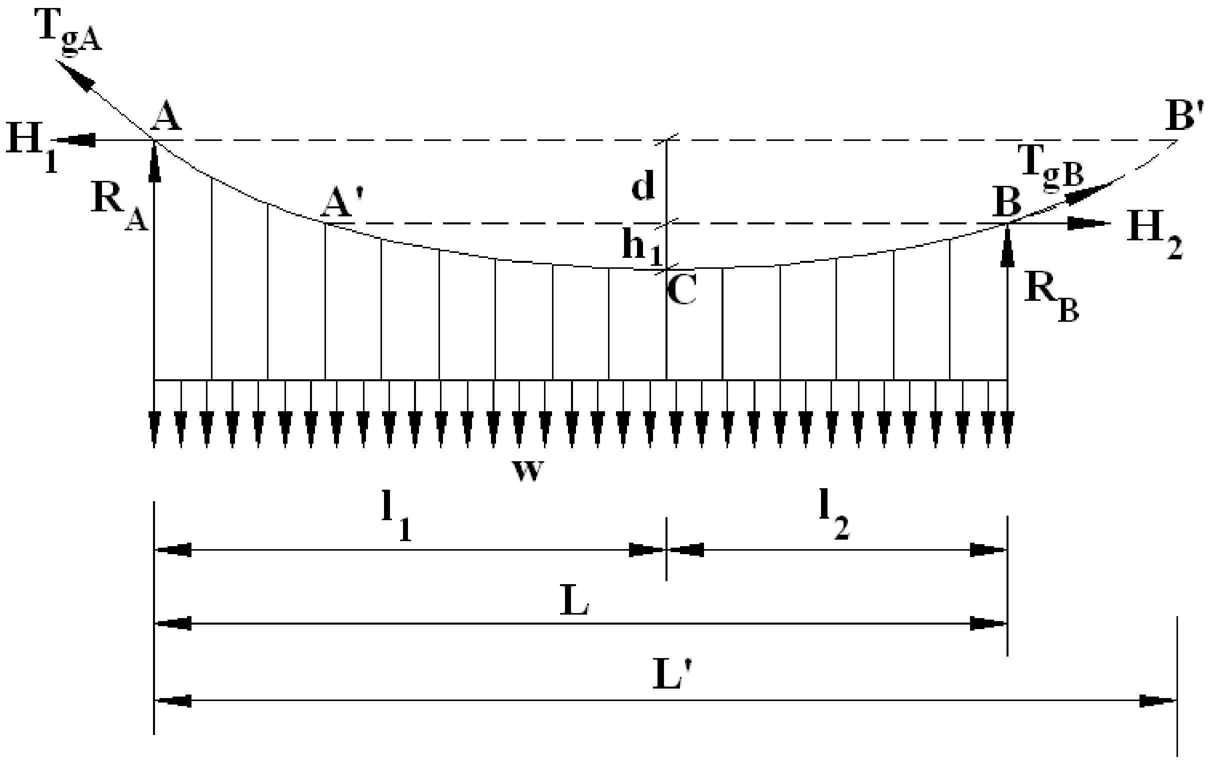
$$Hh + \left(\frac{wL}{2}\right)\left(\frac{L}{4}\right) - R_A\left(\frac{L}{2}\right) = 0$$

$$H = \frac{wL^2}{8h}$$



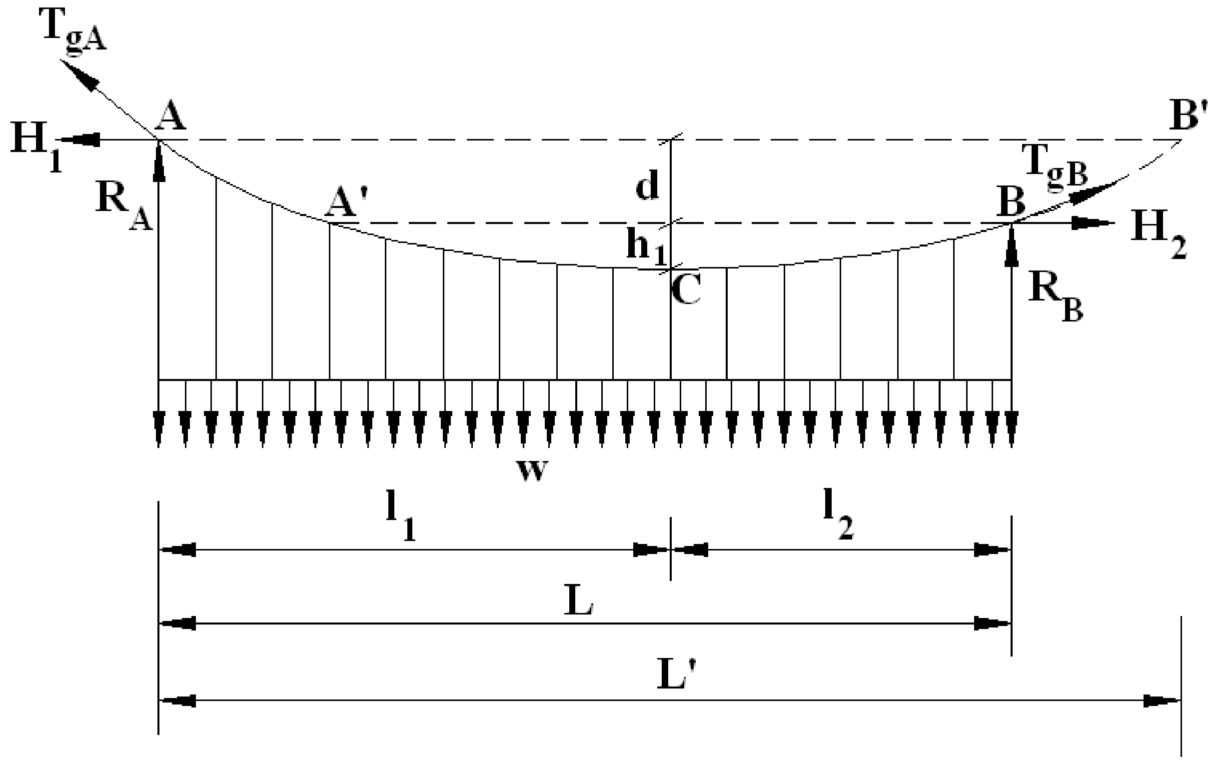
Maximum tensile force in the cable, $T_{max} = \sqrt{R_A^2 + H^2}$

Cable supported at different level



Consider cable ACB':

$$H_1 = \frac{wL'^2}{8(h_1 + d)} = \frac{w(2l_1)^2}{8(h_1 + d)} = \frac{wl_1^2}{2(h_1 + d)}$$



Consider cable A'CB:

$$H_2 = \frac{w\left(2l_2\right)^2}{8h_1} = \frac{w l_2^2}{2h_1}$$

But $H_1 = H_2 = H$, then

$$\frac{w l_1^2}{2(h_1 + d)} = \frac{w l_2^2}{2 h_1}$$

$$\frac{l_1}{l_2} = \sqrt{\frac{h_1 + d}{h_1}}$$ ← Lowest position equation

Vertical Reactions R_A and R_B

$$\sum M_B = 0: \qquad R_A = \frac{wL}{2} + \frac{Hd}{L}$$

$$\sum M_A = 0: \qquad R_B = \frac{wL}{2} - \frac{Hd}{L}$$

Tensile force in the cable at *A* and *B*

$$T_{gA} = \sqrt{R_A^2 + H^2} \qquad T_{gB} = \sqrt{R_B^2 + H^2}$$

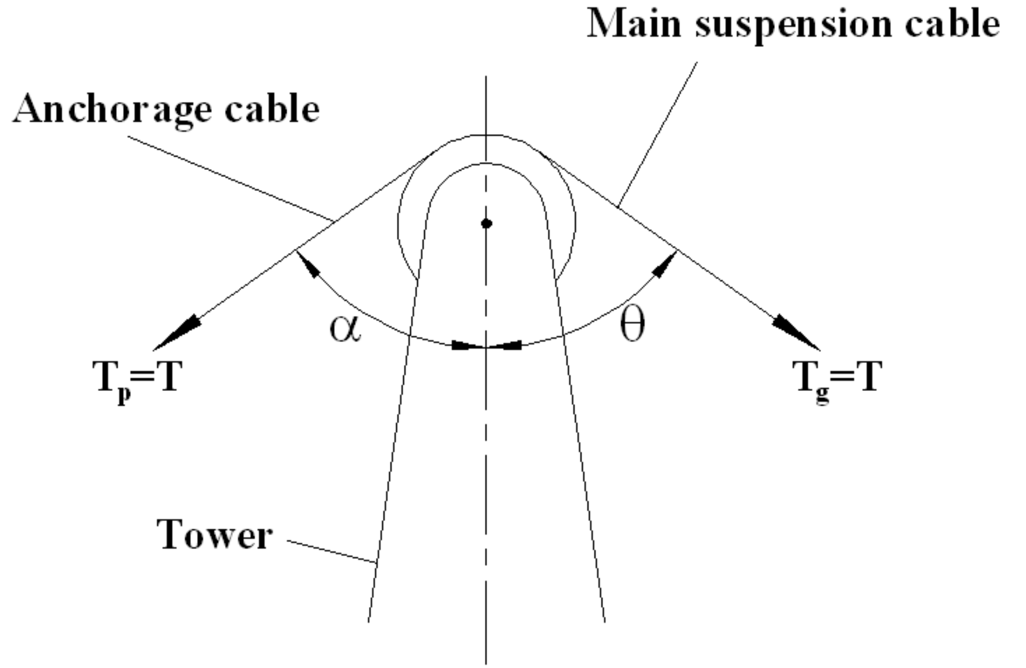
CABLE SUPPORT

Two types – *Sokong Takal Pandu*, Roller Support

Sokong Takal Pandu ← Driven pulley support

Smooth Pulley:

$T_p = T_g = T$



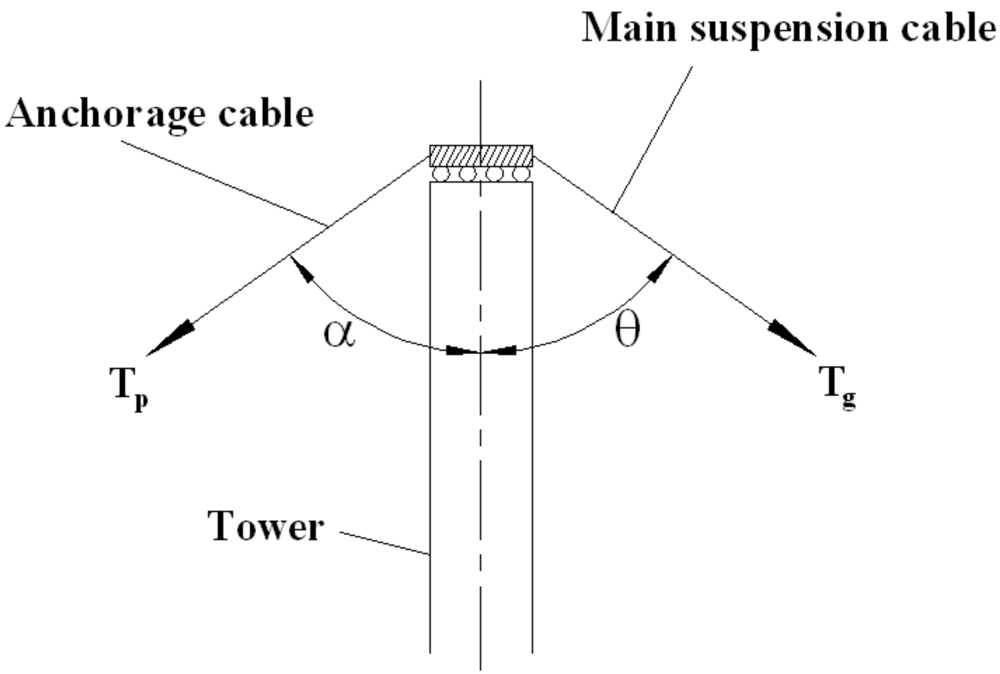
Reactions from tower (*menara*):

$$R_m = T(\cos\alpha + \cos\theta)$$

$$H_m = T(\sin\alpha - \sin\theta)$$

Menara

Roller Support



Note: $T_p \neq T_g$

To ensure the roller not moves laterally:

$$T_p \sin\alpha = T_g \sin\theta$$

Equilibrium Vertically:

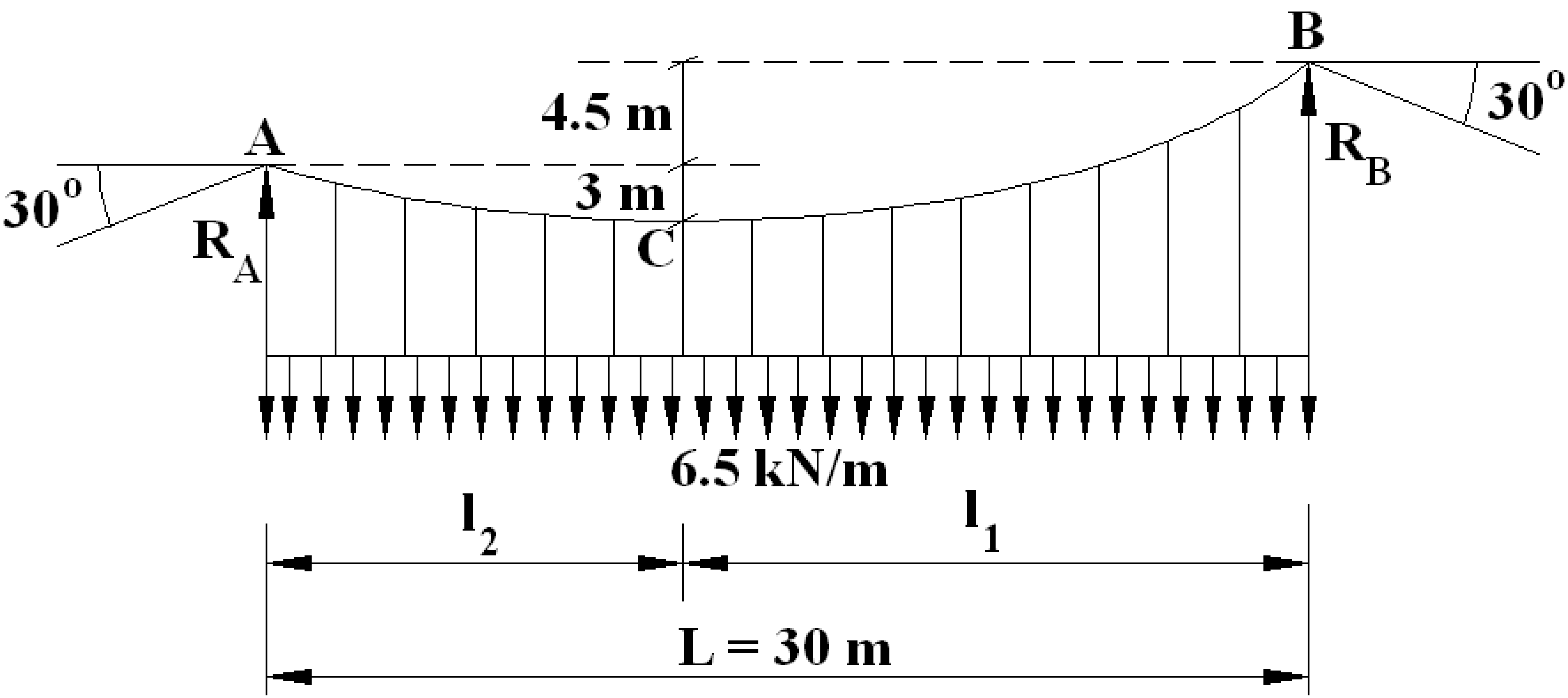
$$R_m = T_p \cos\alpha + T_g \cos\theta$$

Example 7.1

Figure below shows a cable structure supported by frictionless roller which is located on top of tower. Determine:

(a) Maximum and minimum tensile force exerted by the cable

(b) Vertical reaction force exerted by tower at *A* and *B*



Solution

Find l_1 and l_2

$$\frac{l_1}{l_2} = \sqrt{\frac{3+4.5}{3}} = 1.581 \qquad (a)$$

$$l_1 + l_2 = 30 \qquad (b)$$

Solving equations (a) and (b) give l_1 = 18.376 m and l_2 = 11.623 m

Find H

$$H = \frac{wL^2}{8h} = \frac{w(2l_2)^2}{8(3)} = \frac{6.5(2\times 11.623)^2}{24} = 146.35\,kN$$

or

$$H = \frac{wL^2}{8h} = \frac{w(2l_1)^2}{8(3+4.5)} = \frac{6.5(2\times 18.376)^2}{8(3+4.5)} = 146.33\,kN$$

Take H = 146.35 kN

Find R_A and R_B in the cable

$$\sum M_B = 0: \qquad R_A = \frac{wL}{2} - \frac{Hd}{L} = \frac{6.5(30)}{2} - \frac{146.35(4.5)}{30} = 75.5\,kN$$

$$\sum M_A = 0: \qquad R_B = \frac{wL}{2} + \frac{Hd}{L} = \frac{6.5(30)}{2} + \frac{146.35(4.5)}{30} = 119.5\,kN$$

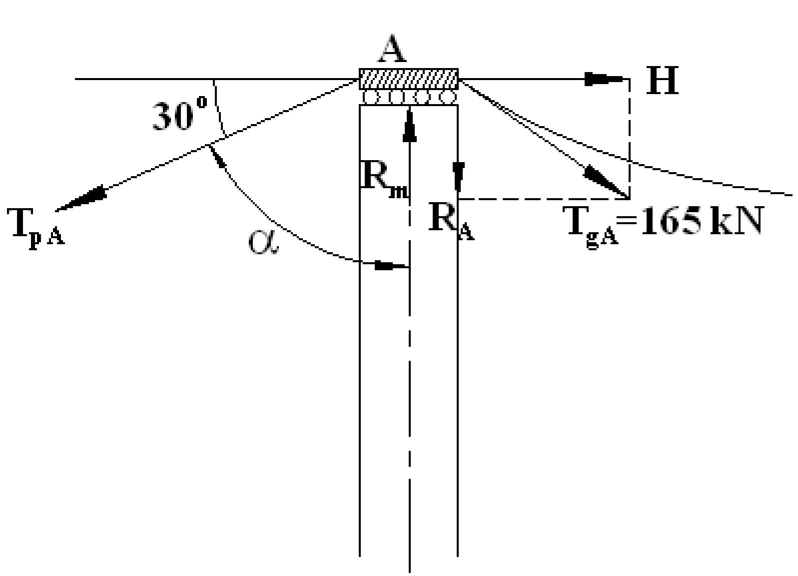
Find tensile force exerted by the cable (at A and B)

$$T_{gA} = \sqrt{R_A^2 + H^2} = \sqrt{75.5^2 + 146.35^2} = 165\,kN$$

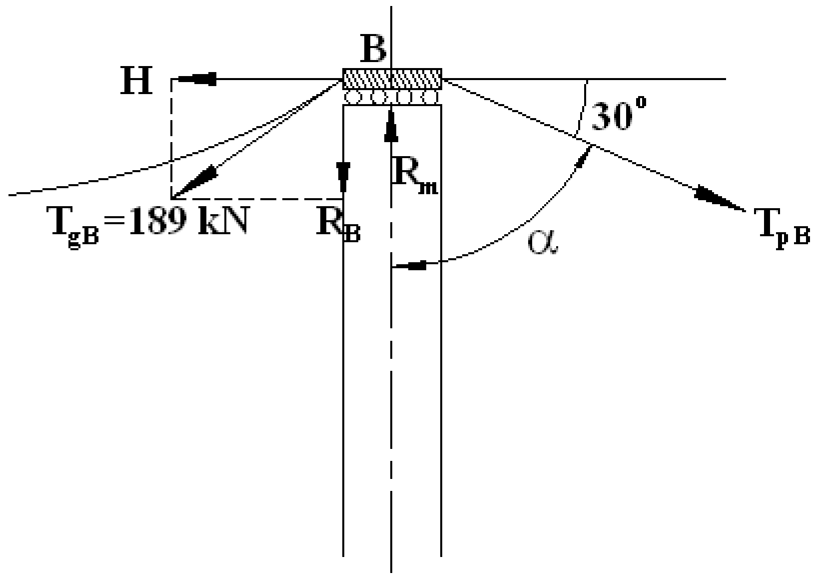
$$T_{gB} = \sqrt{R_B^2 + H^2} = \sqrt{119.5^2 + 146.35^2} = 189\,kN$$

Maximum tensile force occurred at *B*, minimum tensile force occurred at *C* (lowest point in the cable) (=*H*=146.35 kN)

(b) Reaction forces exerted by the tower



(a) Roller support at A



(a) Roller support at B

$$\sum F_x = 0: \quad T_{pA} \sin\alpha = H$$

$$T_{pA} \sin\left(90^\circ - 30^\circ\right) = 146.35$$

$$T_{pA} = 169\, kN$$

$$\sum F_y = 0: \quad R_m = T_{pA} \cos\alpha + R_A$$

$$= 169 \cos 60^\circ + 75.5$$

$$= 160\, kN$$

$$\sum F_x = 0: \quad T_{pB} \sin\alpha = H$$

$$T_{pB} \sin\left(90^\circ - 30^\circ\right) = 146.35$$

$$T_{pB} = 169\, kN$$

$$\sum F_y = 0: \quad R_m = T_{pB} \cos\alpha + R_B$$

$$= 169 \cos 60^\circ + 119.5$$

$$= 204\, kN$$