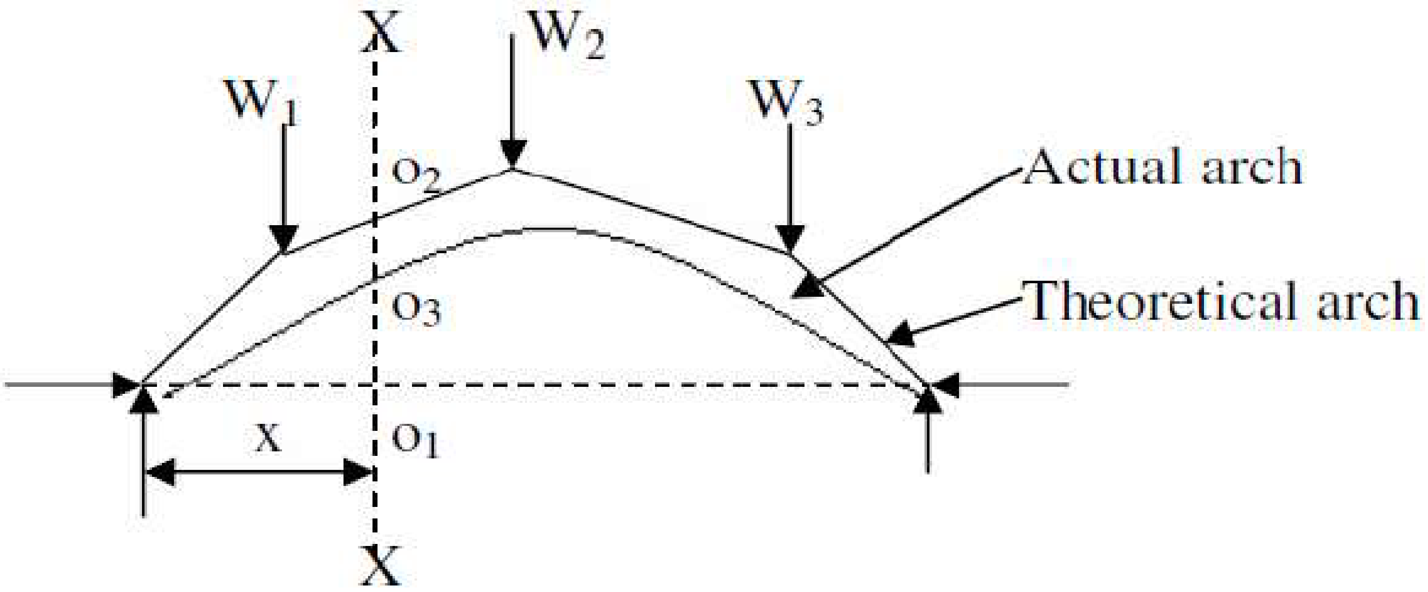
EDDY'S THEOREM

Eddy's theorem states that "The bending moment at any section of an arch is proportional to the vertical intercept between the linear arch (or theoretical arch) and the centre line of the actual arch."

BM_x = Ordinate O_2O_3 x scale factor



THREE HINGED ARCH

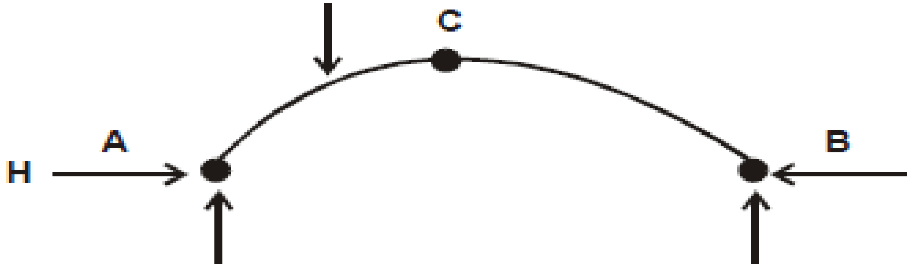
These are Curved Structures which are in use since ancient times. These were mostly used in buildings and the abatements used to be very thick. As our analysis capacity increased due to faster computers, it is now possible to understand behaviour of arches for various support, load and material conditions. These days arch bridges either in Reinforced concrete or the pre-stressed concrete are becoming a common sight due to asthetics of curved surfaces.

Arches when loaded by gravity loads, exhibit appreciable compressive stresses. At supports, horizontal reaction (thrust) is also developed which reduces the bending moment in the arch.

Aches can be built in stone, masonry, reinforced concrete and steel. They can have a variety of end conditions like three hinged arches, two hinged arches and find arches. Considering the geometry these can be segmental, parabolic and circular. An arch under gravity loads generally exhibits three structural actions at any cross-section within span including shear force, bending moment and axial compressive force. The slope of centerline of arch keeps on varying along span so above mentioned three structural actions also vary along span.

If an arch contains three hinges such that two hinges are at the supports and the third one anywhere within span, it is called a three hinged arch. This type of arch is statically determinate wherein reactions, horizontal thrust and all internal structural actions can be easily determined by using the laws of equilibrium and statics. If the third hinge is provided at the highest point, it is called crown of the arch.

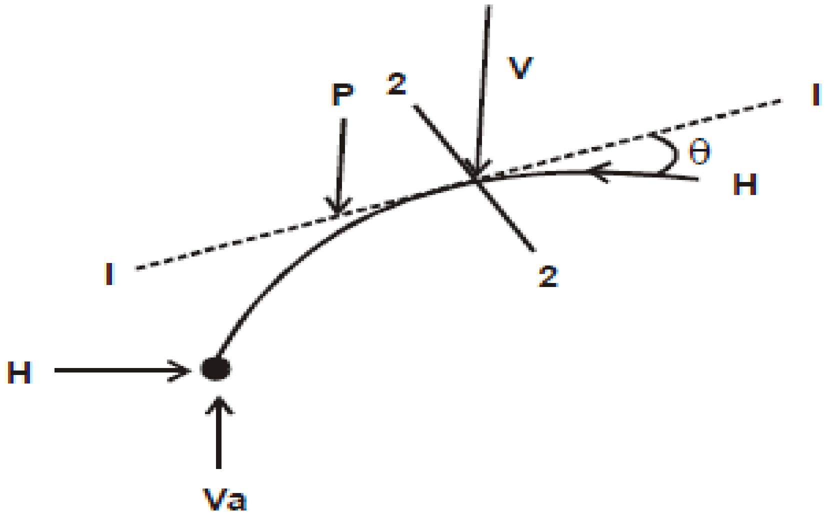
Consider a three hinged arch with third hinge at the crown, then



$M_x = \mu X - Hy$ (1) becomes at center

$M_c = \mu c - Hy_c = 0$

SO $H = \frac{\mu_c}{Y_c}$ (2)



Cutting the arch as shown, and projecting forces along axis 1–1 and 2–2 and putting $V = V_a - P_1$ we have.

$P = H\cos\theta + V\sin\theta$ (3) along 1–1

$Q = H\sin\theta - V\cos\theta$ (4) along 2 – 2

PARABOLIC ARCH

If a three-hinged parabolic arch carries udl over its span, the arch will carry pure compression and no SF or BM. This is because the shape of linear arch (BMD due to loads) will be the same as shape of actual arch.

For a parabolic arch having origin at either of springings, the equation of centre line of arch at a distance X from origin where rise is y will be.

$$y = C.X\,(L - X) \qquad (5)$$

constant C will be evaluated from boundary conditions.

at $X = \frac{L}{2}$, $y = y_c$, we get

$$Y_c = C.\frac{L}{2}.\frac{L}{2} \qquad \text{or} \qquad C = \frac{4\,y_c}{L^2}$$

So

$$y = \frac{4\,y_c}{L^2}\;.\;X\,(L - X) \qquad (6)$$

The slope θ can be calculated from

$$\frac{dy}{dX} = \tan\theta = \frac{4\,y_c}{L^2}(L - 2X) \qquad (7)$$

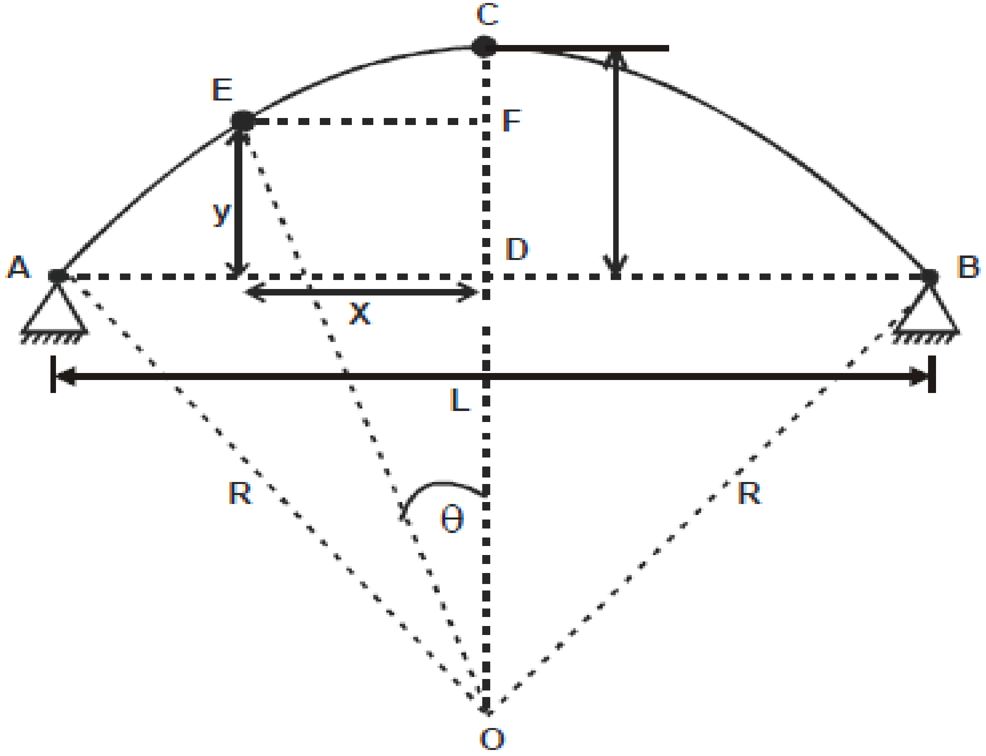
11.4. Circular Arch:

If arch is a part of Circle, it is convenient to have origin at the centre.

Consider triangle OEF

$$OE^2 = EF^2 + OF^2$$

Or $$R^2 = X^2 + (R - y_c + y)^2 \qquad (8)$$



and we also have from triangle ADO

$$\frac{L^2}{4} + (R - y_c)^2 = R^2$$

$$y_c\,(2R - y_c) = \frac{L^2}{4} \qquad (9)$$

As span and central rise are usually known, Radius of arch R can be calculated from (9)

Equation (8) can bge written as $y = \sqrt{R^2 - X^2} - (R - y_c)$

THANKING YOU