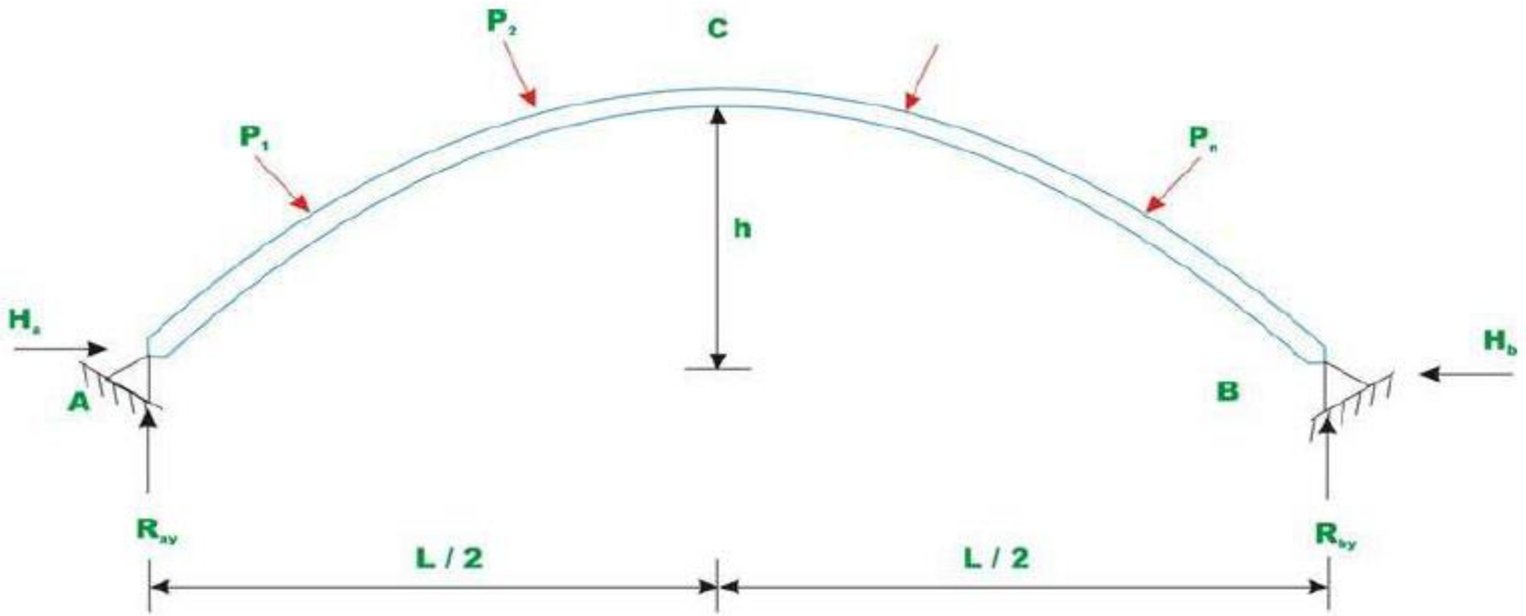
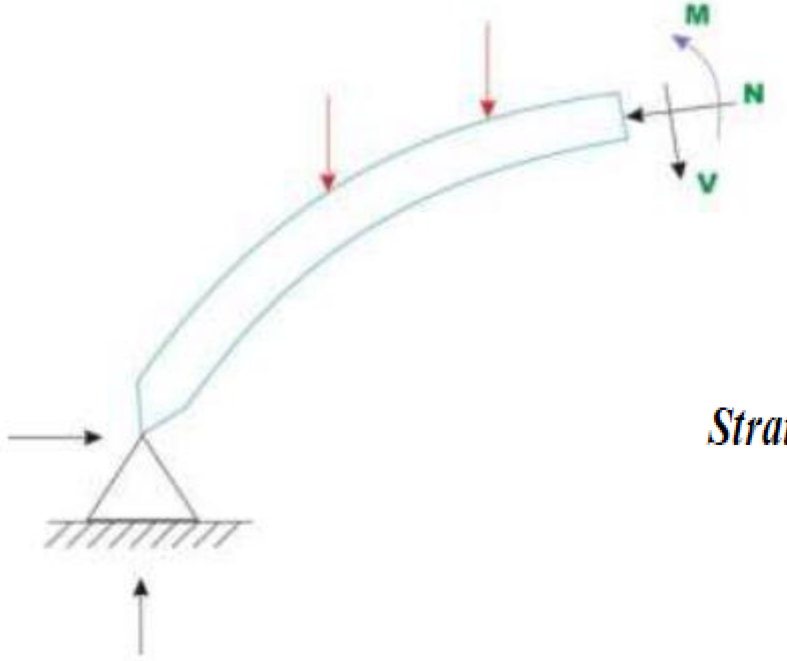
TWO HINGED ARCH

- A typical two-hinged arch is having four unknown reactions, but there are only three equations of equilibrium available.
- Hence, the degree of statically indeterminacy is one for two-hinged arch.





Strain energy due to bending (U_b)

$$U_b = \int_0^s \frac{M^2}{2EI} ds$$

Where,

M	=	*Bending moment*
E	=	*Young's modulus of the arch material*
I	=	*Moment of inertia of the arch cross section*
s	=	*Length of the centreline of the arch*

Strain energy due to axial compression (U_a)

$$U_a = \int_0^s \frac{N^2}{2AE} ds$$

Where,

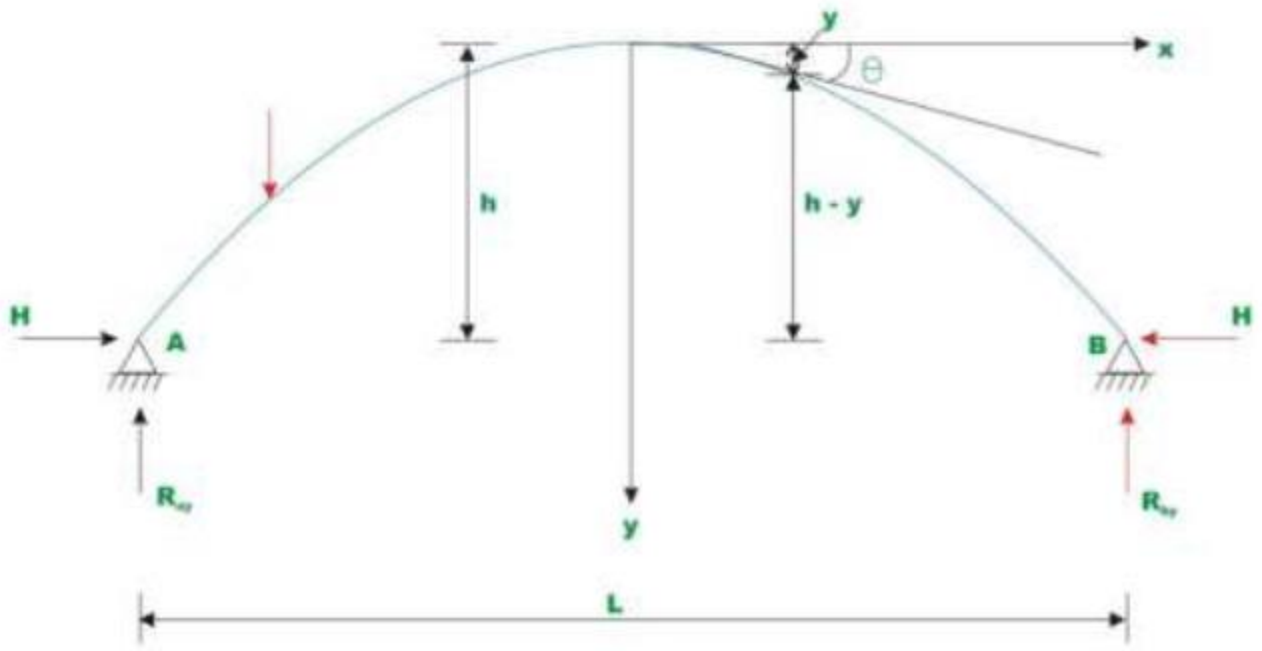
M	=	Bending moment
N	=	Axial compression.
A	=	*Cross sectional area of the arch*
E	=	*Young's modulus of the arch material*
s	=	*Length of the centreline of the arch*

Total strain energy of the arch

$$U = \int_0^s \frac{M^2}{2EI} ds + \int_0^s \frac{N^2}{2AE} ds$$

Symmetrical two hinged arch

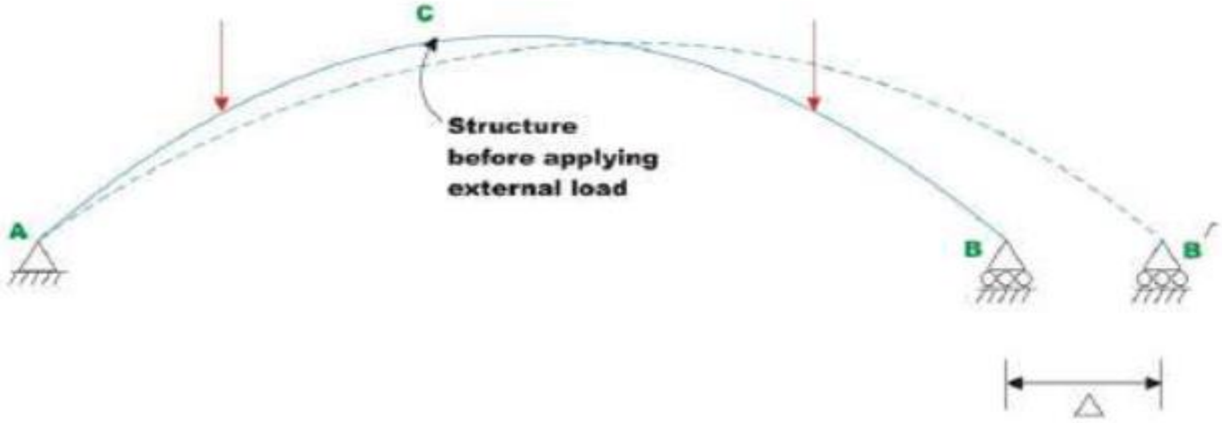
* Consider a symmetrical two-hinged arch as shown in figure.



Let '*C*' at crown be the origin of co-ordinate axes.

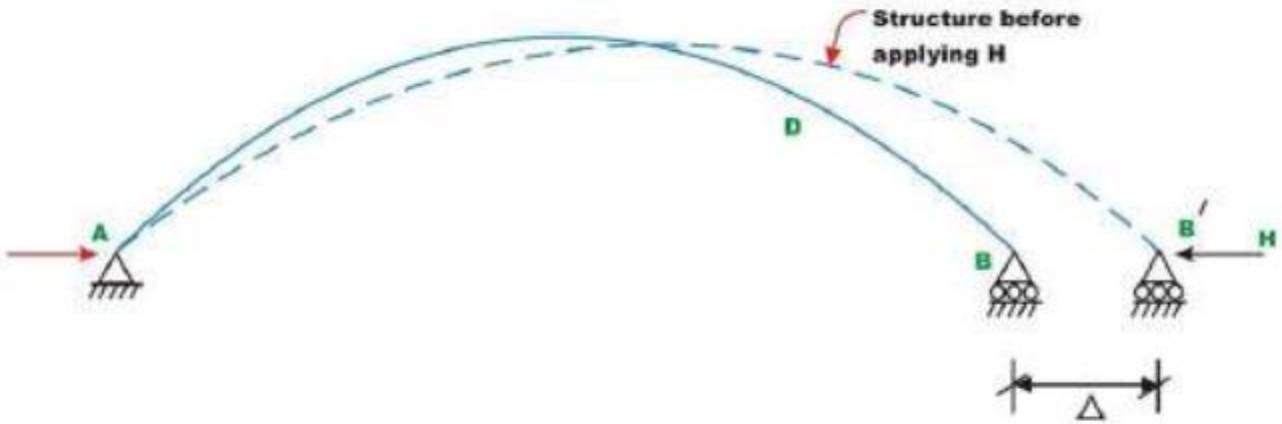
Now, replace hinge at '*B*' with a roller support.

Then we get a simply supported curved beam figure as shown in below.

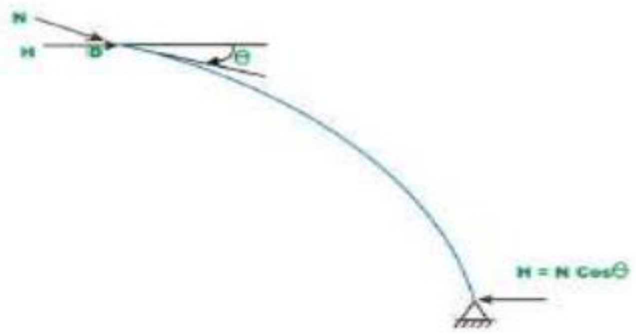


Since the curved beam is free to move horizontally, it will do so as shown by dotted lines. Let M_o and N_o be the bending moment and axial force at any cross section of the simply supported curved beam.

Since, in the original arch structure, there is no horizontal displacement, now apply a horizontal force 'H' as shown in figure.



The horizontal force '***H'*** should be of such magnitude, that the displacement at '*B'* must vanish.



Bending moment at any cross section of the arch **M = M_o – H (h-y)**

The axial compressive force at any cross section **N = N_o + H cos*θ***

Where,

θ ⟶ the angle made by the tangent at D with horizontal

Substituting the value of *M* and N in the equation

$$\frac{\partial U}{\partial H} = 0 = -\int_0^s \frac{M_0 - H(h-y)}{EI}(h-y)ds + \int_0^s \frac{N_0 + H\cos\theta}{EA}\cos\theta\, ds$$

$$H = \frac{\int_0^s M_0 \tilde{y}\, ds}{\int_0^s \tilde{y}^2\, ds}$$

Temperature effect

Consider an unloaded two-hinged arch of span L.

When the arch undergoes a uniform temperature change of $T\,°C$, then its span would increase by $\alpha\, L\, T$ if it were allowed to expand freely.

α is the co-efficient of thermal expansion of the arch material.

Since the arch is restrained from the horizontal movement, a horizontal force is induced at the support as the temperature is increased.